Ask a Scientist Pi Day Puzzle Party
3.14 2017
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1. LONG SEQUENCE

A sequence of numbers is created by following these three rules:

- The first number in the list is 12
- If a number in the list is even, divide it by two to make the next number in the list
- If a number in the list is odd, multiply it by three then add one to make the next number in the list

So, following these rules, the first few terms of this sequence are: 12, 6, 3, 10, 5... and so on.

QUESTION: What's the sum of this sequence's first 2017 terms?

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2. THE PENNY GAME

Donald Trump brings you a bucket filled with pennies and challenges you to the following game: you will each take turns putting a penny on a circular table. Once a penny is placed, you can't move it or put a penny overlapping with it. The last person who is able to place a penny on the table wins. You're given the choice of going first or second.

QUESTION: Explain your winning strategy.

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3. THE INFINITE ENEMY

The enemy's submarine is traveling on the number line (which extends infinitely in both directions and is underwater). We know that the sub is traveling at a constant speed and that at 10 o'clock every day it is at an integer. However, we do not know where the sub started, what speed it is traveling, or even which direction. You can fire one perfectly accurate missile at 10am every day.

QUESTION: What is the strategy you will use to (eventually) hit the sub?

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4. STRANGE QUARTERS

Using at most three straight cuts (and no measuring instruments), cut a standard rectangular sheet of paper into four pieces, each of which is a different shape, but all of which have exactly the same area.

Oh, by the way, none of your cuts may be parallel to the edges of the paper.

QUESTION: Draw a diagram showing your strategy.

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5. DOUBLE ZERO SUM GAME

Consider the following two-player game: Starting with the number 0, players take turns adding to the current sum; on your turn, you can add either 4 or 7. If on your turn you can make the new sum end in two zeros (i.e., if your turn leaves a multiple of 100), you win.

QUESTION: Assuming best play, is there a winning strategy for either player, or will the game go on indefinitely? If there is a winning strategy, should you move first or second, and how do you play from there?

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6. THREE MATHEMATICIANS

One day the king of Flatland realized that he needed a mathematician to help him solve problems in the kingdom. So he called his advisors together and asked them to find the best mathematician in the kingdom. A week later, the advisors returned with three mathematicians. The advisors agreed that these were clearly the best mathematicians in the kingdom but they could not decide which one to pick. The king thought for a minute and then said something to a servant who left and returned with 6 hats (3 red and 3 white). The mathematicians were blindfolded and a hat placed on each of their heads.

The king said, “This is how I will choose my official mathematician. You each have a red or white hat on your head. We will remove the blind-folds and whoever is the first person to correctly identify the color of her own hat will be the chosen mathematician.” The king also told them that any attempt at communication would lead to their immediate expulsion from the contest. Further, to eliminate any guessing, he told the mathematicians that if they incorrectly identified the color, they would be required to spend the rest of their lives grading math exams, 12 hours a day, 7 days a week. Finally, he told the mathematicians that at least one of them was wearing a red hat and ordered the blindfolds removed. As it turns out, they all had a red hat … so the information didn’t seem to be of much use. However, after a few minutes, one of the mathematicians stood up and correctly declared that she was wearing a red hat. She became the official mathematician of the kingdom and the other two returned to their university positions.

QUESTION: How did she know for sure that she was wearing a red hat?

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7. DIVISIBLE BY 7

Let A, B, and C be distinct digits, and let ABC and CBA be three-digit integers that are both divisible by 7.

(Assume that leading zeroes are not allowed; i.e., 70 cannot be considered to be a three-digit integer by writing it as “070”.)

QUESTION: Please find the sum of all such possible numbers ABC.

SOLUTIONS

→

1. LONG SEQUENCE

Answer = 4750

The pattern is: 12 6 3 10 5 16 8 4 2 1 4 2 1 4 2 1 4 2 1 4 2 1 ...

So: 60 + (2010 / 3) * 7 = 4750

2. THE PENNY GAME

Place the penny in the center of the table. (If it's a very small table, you've already won!)

The second player places a penny some place.

You place your penny in the mirror reflection (across the center) from where your opponent's last one was placed.

Continuing in this manner, player 2 eventually runs out of places to put a penny and you win.

3. THE INFINITE ENEMY

First, note that since the sub is at an integer location every day, it must have started from an integer location and must be travelling at an integral speed.

Suppose that today the sub is at integer "a" and is traveling at a velocity of "b" integers per day. Now, a could be negative, 0, or positive; and so can b, but nonetheless both are integers. We can enumerate all possible pairs (a,b) by imagining a spiral in the two-dimensional lattice that starts at (0,0) and works its way outward:

(0,0) ... (1,0) ... (1,1) ... (0,1) ... (-1,1) ... (-1,0) ...

Every pair of integers is included in this spiral, so clearly the actual (a,b) must be as well. We just need to work our way through this list, firing each day at 10am at the location where the enemy would be if that pair of integers were the true (a,b).

So, for example:

On day 0:
fire at 0 (sub's location today, if a = 0 and b = 0)

On day 1:
fire at 1 (sub's location tomorrow if a = 1 and b = 0)

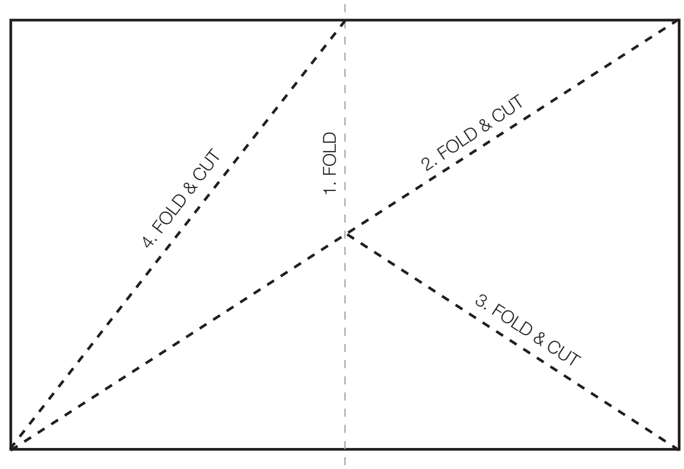
On day 2:
fire at 3 (sub's location day-after-tomorrow if a = 1 and b = 1)

... and so on.

Eventually we are guaranteed to try for the correct (a, b), and on that day, we'll hit the sub.

4. STRANGE QUARTERS

There are multiple solutions; here is one:



5. DOUBLE ZERO SUM GAME

The obvious first thought is to go to 1100, but you hit 400 or 700 first.

As the first player, start off by playing a 4. After that, make sure the sum keeps leaping in increments of 11 by doing the opposite of your opponent's every play (i.e., if he plays a 4 then you play a 7; if he plays a 7 then you play a 4).

So the pattern would go:

4 (+11) **15** (+11) **26** (+11) **37**...

You'll eventually get to 400, which is a multiple of 11 (396) plus the original 4 that you played.

6. THREE MATHEMATICIANS

By definition not all the hats are white. If exactly two hats were white, the person with a red hat would see the white hats, know hers was red and immediately stand up. Since no one does that, they all know that there is at most one white hat. They all look and see two red hats and think, "Suppose I have a white hat. The other two would see that…and knowing there was at most one white hat, they would have stood up and proclaimed their hats as red. Since no one did, I must in fact also have a red hat." The woman who stood up first, made the deduction a bit quicker than the other two.

7. DIVISIBLE BY 7

Answer = 2240

ABC can be written 100a + 10b + c.

If (100a + 10b + c) and (100c + 10b + a) are both to be divisible by 7, then so must be their difference: 99(a - c)

Let's look at 99(a-c). For that to be divisible by 7, either 99 or (a-c) must be divisible by 7. Since 99 is not divisible by 7, we need (a-c) to be divisible by 7.

Therefore a and c must be either 9 & 2, or 8 & 1, or 7 & 0. If either a or c is zero, then we get something that isn't a three-digit integer, so reject the pair 7 & 0, leaving only 9 & 2, and 8 & 1. Each such pair can go in either order, and each such pair works with only one value of b.

That gives us the four numbers 952, 259, 861, and 168, which add to the answer: 2240