Ask a Scientist Pi Day Puzzle Party
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1. THE SPOTTY ORANGE

An orange has 5 tiny spots on its skin. Can you cut the orange into two approximately equal pieces (with a single straight cut) so that one half has no more than one spot on it?

QUESTION: Describe how you would do this. Your solution must work for all configurations of spots on the skin.

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2. POST OFFICE

March 14 is a busy day for the postal service because of all the Pi Day cards that are sent. One year, Postal Worker Circum, working by himself, was able to get through all of the mail in eight hours. The following year, Postal Worker Ference, working by herself, was able to get through all of the mail in just four hours.

QUESTION: This year, Postal Workers Circum and Ference will be working together. Assuming they are individually no faster or slower than in previous years and assuming that every March 14 has the same amount of mail, how long will it take them to get through the mail working together?

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3. WHAT COMES AFTER 400

QUESTION: What number comes next in this series?

100

121

144

202

244

400

?

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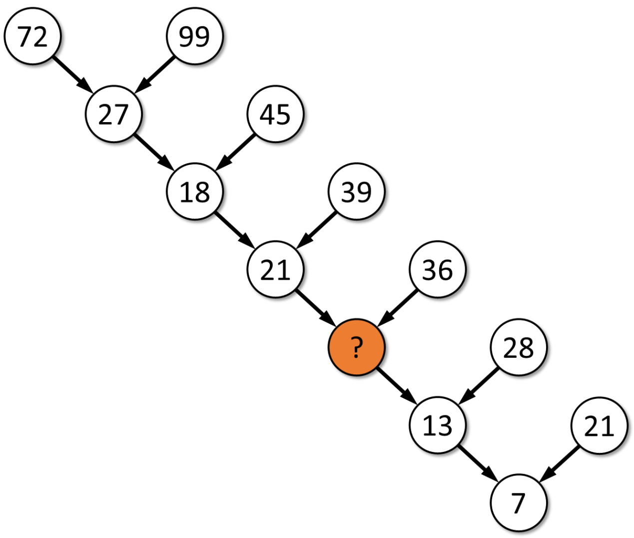
4. FACTORIAL PRODUCT!

Imagine multiplying 99 of the first 100 factorials together. In other words, you take 1! x 2! x 3! x ... x 99! x 100!, but you skip one of them.

QUESTION: Which one should you skip so that the final product will be a perfect square?

5. THE MISSING NUMBER

QUESTION: What is the missing number in this diagram?



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6. RUTHLESS, RATIONAL PIRATES

There are 1000 pirates who are all extremely greedy, heartless, and perfectly rational. They're also aware that all the other pirates share these same characteristics. They're all ranked by the order in which they joined the group, from Pirate One down to Pirate One Thousand.

The pirates have stumbled across a huge horde of treasure, and they have to decide how to split it up. Every day they will vote to either kill the lowest ranking pirate, or split the treasure up among the surviving pirates. If 50% or more of them vote to split it, the treasure gets split. Otherwise, they kill the lowest ranking pirate and repeat the process, until half or more of the pirates decide to split the treasure.

QUESTION: At what point (that is, when how many pirates are left alive) will the treasure be split?

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7. PALINDROMES

A five-digit palindrome is a positive integer with respective digits abcba, where a is not zero. Let S be the sum of all five-digit palindromes.

QUESTION: What is the sum of the digits of S?

SOLUTIONS

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1. THE SPOTTY ORANGE

Pick any two points (it may be obvious which to pick, but if not, just pick any). Cut the great circle whose equator they both lie on, with both points just barely to one side or the other.

If you wind up with 5 on one hemi and 0 on the other… GOOD!

If you wind up with 4 on one hemi and 1 on the other… GOOD!

If you wind up with 2 on one hemi and 3 on the other, just shift the equator ever so slightly to the other side of the points…. GOOD!

2. POST OFFICE

It will take them <u>2h 40m.</u>

3. WHAT COMES AFTER 400

100 = 100 in base 10
121 = 100 in base 9
144 = 100 in base 8
202 = 100 in base 7
244 = 100 in base 6
400 = 100 in base 5
<u>1210</u> = 100 in base 4

4. FACTORIAL PRODUCT!

Consider what a list of the factors of 1! x 2! x 3! x ... x 99! x 100! would look like (not the smallest prime factors, but 99 twos, 98 threes, 97 fours, etc.)

Looking at this list, it becomes clear that if you omit 2 x 4 x 6 x 8 x … x 100 the result will be a perfect square

But the value of 2 x 4 x 6 x 8 x … x 100 is not a factorial itself, so there must be a smaller number that can also work

2 x 4 x 6 x 8 x … x 100 = 2^{50} x 50!

2^{50} is a square itself, so we can dump that one out and we're left with the answer: <u>you should skip 50!</u>

5. THE MISSING NUMBER

Add all digits of first two numbers to yield the third number.

<u>The missing number is 12.</u>

6. RUTHLESS, RATIONAL PIRATES

Each pirate will vote to kill, until the number of pirates reaches the lowest number for which that pirate would survive. Let's examine everyone's strategy:

Pirate #1 will vote to "share" — i.e. disburse — the treasure when he's the only pirate left, but until then, she'll vote to kill. * Pirate #2 can save himself and get half the treasure by holding off until there are just two pirates left, so he'll vote to kill until then. * Pirate #3 can't survive if there are only 3 pirates left. But when there are 4 left, both #3 and #4 can survive by voting to disburse, so both will do so. For larger numbers, they'll vote to kill, since they have a foolproof plan for getting a quarter of the loot each. So we can think of Pirates #3 and #4 as being in a "voting bloc."

Pirates #5 through #8 are in a voting bloc too. Take a minute to think about this, and let me know if it doesn't seem clear.

Pirates #9 through #16 are in a voting bloc too, as are #17 through #32.

We can keep making voting blocs for every group of pirates whose number ends in a power of two; each such bloc has the unstoppable power to end the voting and force the game to game through disbursement.

<u>Therefore, they will split the treasure when there are exactly 512 pirates left,</u> because 512 is the highest power of two less than the 1000 pirates we started with.

7. PALINDROMES

Note that abcba = a000a+b0b0+c00.

Because 1+2+· · ·+9 = (1/2)·(9·10) = 45, the sum of all integers of the form a000a is 450,045. For each of the nine possible values of a, there are 10·10 = 100 choices for b and c. So the final sum will include 100·450,045 as the contribution from the "a000a" part of the palindromes.

Similarly, the sum of all integers of the form b0b0 is 45,450.
For each value of b there are 9·10 = 90 choices for a and c.
So the final sum will include 90·45,450 as the contribution from the "b0b0" part of the palindromes.

Finally, the sum of all integers of the form c00 is 4500.
For each c there are 9·10 = 90 choices for a and b.
So the final sum will include 90·4500 as the contribution from the "c00" part of the palindromes.

We add those three contributions to get S.
That's 100·450,045 + 90·45,450 + 90·4500.

This sums to 49,500,000.

<u>So the answer is that the sum of the digits of S is 18</u>.