Ask a Scientist Pi Day Puzzle Party
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1. PHONE DROP

Congratulations! You've just been hired as an intern, working for an iPhone case reseller. The company has just received two identical samples of the latest model, which the manufacturer says is completely indestructible.

But Engineering has already reviewed the samples, and concluded that the new model will not survive a 101-story drop.

Marketing is eager to know: what's the highest story it *can* be dropped from, and still survive? Your mission is to research the answer to that question, starting tomorrow at 9am.

You'll arrive at company headquarters, which is exactly 100 stories tall. You'll then execute a series of test drops from whichever floors you choose. You figure it will take you twenty minutes to set up and execute each drop (counting the time to film it for YouTube, naturally). You will work without breaks of any kind, as is the tradition for interns at this company.

Your best friend plans to pick you up at the office when you're done with your first day, so that you can both go out and celebrate your new job. But your friend can't stand waiting around.

QUESTION: What time should you have your friend arrive at work, so that you can be 100% sure that you'll be finished? (Note: you must choose the earliest possible time in order to receive credit for this answer. It wouldn't be right to sacrifice valuable celebration time!)

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2. LICENSE PLATES

Consider the most common type of California license plate, which consists of a number followed by three letters followed by three numbers.

Sometimes one sees a car with such a plate on which the same letter appears exactly twice. More rarely, a plate with exactly three of the same letter is seen.

Call the first category "double plates" and the second "triple plates."

QUESTION: How many times more double plates are there than triple plates?

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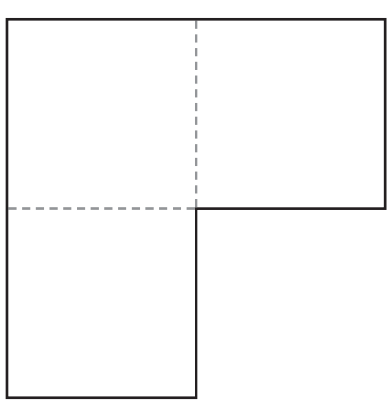
3. SQUARE DIVISION

Here is a square that's had its bottom right quarter removed. The resulting shape can easily be divided into three identical squares, as shown here.

QUESTION: Divide the resulting shape into four identical shapes instead of three.



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4. THRICE PROPER

Let's agree that a **divisor** of a number n is, by definition, a positive integer that evenly divides n. So, for example, the divisors of 4 are 1, 2, and 4.

Let's agree that a **proper divisor** is any divisor of the number, other than the number itself. So, for example, the proper divisors of 4 are 1 and 2 only.

QUESTION: How many positive integers have exactly three proper divisors, each of which is less than 50?

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5. UNFAIR COIN

You and your archenemy sit across from each other at a mahogany table. Between you lies the prize you've been fighting over all these years: a million-dollar diamond. Only one of you can take it; it cannot be split. The only resolution you can agree upon is to flip a coin to ensure that you each have an equal chance of getting the jewel.

However, the only coin available is known to both of you to be biased in favor of heads.

"That won't matter," says your archenemy, "because coin flips can guarantee a fair outcome even when the coin is weighted, as you surely are aware."

QUESTION: Are you "surely aware"? Explain a method by which you'll each have a 50% chance of winning, and thereby keep your archenemy from embarrassing you through superior knowledge and/or repartee.

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6. THE BRIDGE

Suppose you stand in middle of the Golden Gate Bridge and point one arm directly toward north pole, and point your other arm directly toward the south pole. (Note: "directly" doesn't mean along the surface of the globe.)

QUESTION: To the nearest degree, what angle will your two arms make?

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7. HATS AND MATHEMATICIANS

You are part of a group of thirty mathematicians, imprisoned for proclaiming the "heresy" that 2 + 2 = 0 (in mod 4). The prison warden, a friendly chap, has offered you a sporting chance at escape. So today, a red or black hat was placed on each of your heads. You are guaranteed that your hat is one of these two colors. You all were told that any attempt to physically determine the color of one's own hat will result in immediate death by sniper.

Starting tomorrow, any prisoner wearing a black hat may walk out the gate unharmed. But, any prisoner wearing a red hat who attempts to leave will be shot.

Tonight, the prison chaplain, in a merciful mood, informed you all that at least one of your hats is black. (What he did not say is that in fact *all* of your hats are black.)

You are never allowed to communicate with each other in any way. None of you will risk leaving unless you are are certain that you are wearing a black hat.

QUESTION: Will any of you ever leave? If so, who, and when?

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8. GOLD AND SILVER COINS

Robert has 4 indistinguishable gold coins and 4 indistinguishable silver coins. Each coin has an engraving of a face on one side, but not on the other. He wants to stack the eight coins on a table into a single stack so that no two adjacent coins are face to face.

QUESTION: How many possible distinguishable arrangements are there of the 8 coins.

SOLUTIONS

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1. PHONE DROP

Answer: 1:40pm

14 drops * 20 minutes/drop = 280 minutes

9am + 280 minutes = 1:40pm

2. LICENSE PLATES

Answer: 75

There are 17,576 possible letter combos (26*26*26).

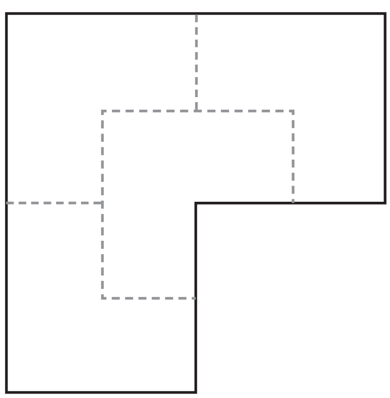
15600/17576 will have distinct letters (26*25*24)

1950/17576 will have a double ((26*25)*3 ways to order the distinct ones)

26/17576 will be a triple

The ratio of double letters to triple letters is 1950/26 which equals 75.

3. SQUARE DIVISION



4. THRICE PROPER

Answer: 109

There are two types of integers n that have three proper divisors. If n = pq, where p and q are distinct primes, then the three proper divisors of n are 1, p, and q; and if n = p^3, where p is a prime, then the three proper divisors of n are 1, p, and p^2. Because there are 15 prime numbers less than 50, there are 15c2 = 105 integers of the first type. There are 4 integers of the second type because 2, 3, 5, and 7 are the only primes with squares less than 50. Thus there are 105 + 4 = 109 integers that meet the given conditions.

5. UNFAIR COIN

Answer: Both people flip the coin once each. If they both get heads or both get tails, dismiss and repeat. When there's finally a mismatch, the person with heads wins. (Or one person could do both flips as long as they work out ahead of time that HT means you win and TH means enemy wins.)

6. THE BRIDGE

Answer: 90°

No matter where you are, except at the poles, the angle will always be 90° (like right triangles inscribed within a circle).

7. HATS AND MATHEMATICIANS

Answer: They all leave on Day 30

On day 1, they all ask, could there be 29 red hats. If someone saw 29 red hats, knowing that at least one was black, that person would have to have a black hat and so would. No one does leave and so everyone knows – they all think logically – that there are at most 28 red hats. On day 2, they ask, could there be 28 red hats. If someone sees 28 red hats, the person would leave. Again no one does, so there must be only 27. And so this continues. On day 29, everyone asks, could there be 1 red hat? Again no one sees one and so no one leaves … and they know that there are no red hats. So on day 30 they all leave.

8. GOLD AND SILVER COINS

Answer: 630

First consider the orientation of the coins. Label each coin U or D depending upon whether it is face up or face down, respectively. Then for each arrangement of the coins, there is a corresponding string consisting of a total of eight U's and D's that is formed by listing each coin's label starting from the bottom of the stack. An arrangement in which no two adjacent coins are face to face corresponds to such a string that does not contain UD. Thus the first U in the string must have no D's after it. The first U may appear in any of eight positions or not at all, for a total of nine allowable strings. For each of these nine strings, there are 8c4 ways to pick the positions for the four gold coins, and the positions of the silver coins are then determined. Thus there are 9*(8c4) = 630 arrangements that satisfy Robert's rules of order.

TIEBREAKER

12 FACTORS

60 has exactly 12 factors. Name another two-digit number with exactly 12 factors.

SOLUTION

72, 84, 90, 96