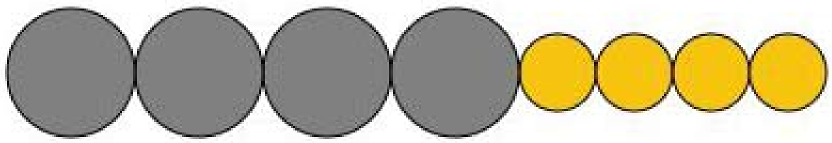
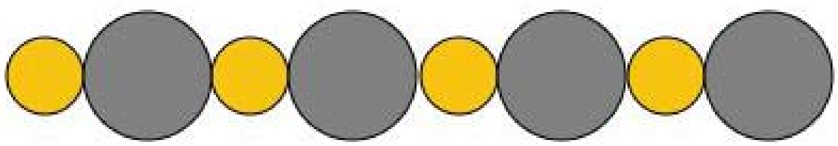
# ASK A SCIENTIST HOLIDAY PUZZLE PARTY
**December 9th, 2007**
http://askascientistsf.com

### 1. Coins

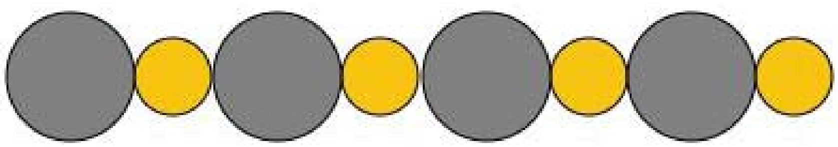
Arrange four quarters and four pennies like so:



Your goal is to get the coins into an arrangement where no quarter touches quarter and no penny touches penny — in only four moves. So your final arrangement will look like this:



Or this:



A “move” consists of sliding any two adjacent coins, together, into a new position along the line. Yes, the two adjacent coins may jump other coins in order to get to their new position — they don’t have to slide along the line unobstructed. No, you may not rotate them as they move — the pair’s left coin must stay on the left, and the right coin must stay on the right. Yes, it’s ok to leave a gap in the line when a pair of coins is moved —but the final arrangement must leave no gaps. It must look just like it does in the examples above.

## 2. Handshakes

I invite ten couples to a party at my house. I ask everyone present, including my wife, how many people they shook hands with. It turns out that everyone questioned — I didn't question myself, of course — shook hands with a different number of people. If we assume that no one shook hands with his or her partner, how many people did my wife shake hands with? (I did not ask myself any questions.)

## 3. Strawberry Ice Cream

I visited a math professor of mine for dinner at his home (well, not really but shh! it's part of the problem!) who had pictures of his three daughters on his mantle. He had had pictures taken of the three girls when each was a particularly adorable age — the same age for all three, as it happens. Unfortunately, this made it impossible for me to determine which was the oldest. So I had to ask him.

Being a math professor, however, he declined to answer directly, telling me only that the product of their current ages was 72. "However," he added, "since that isn't enough information to determine their ages, I'll also tell you that the sum of their ages happens also to be the number of our street address." (Of course, I understood that each daughter's age was to be considered an integer for this puzzle.)

I darted outside to check the number on his mailbox. I was daunted to discover that I still didn't have enough information to determine their ages, and I returned to tell him so.

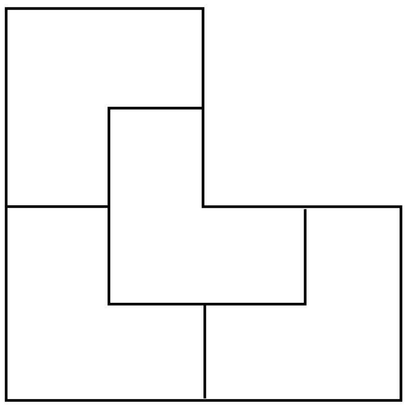
"That is an astute observation," he said, smiling. "So you'll be glad to know that my oldest daughter prefers strawberry ice cream."

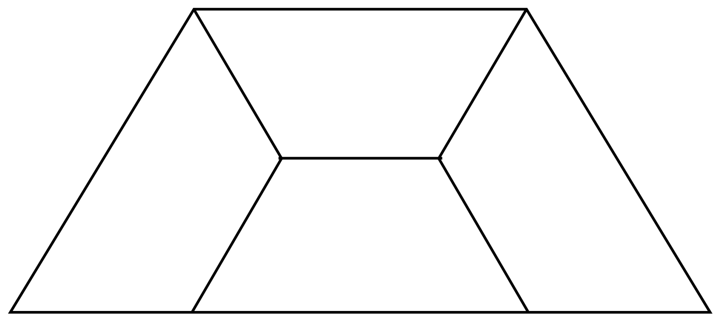
Finally! I knew their ages.

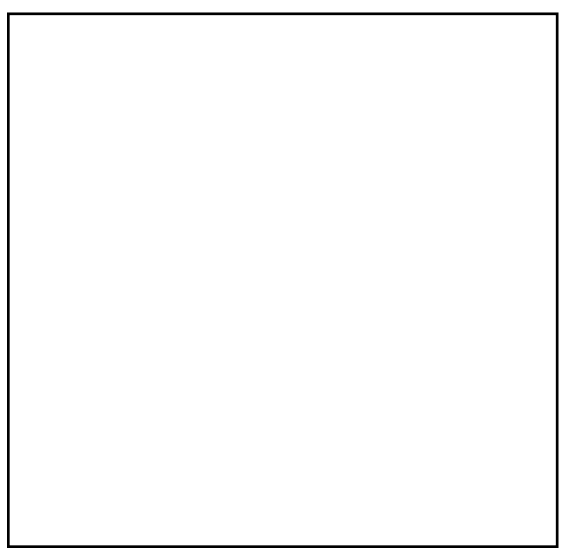
Do you?

## 4. Dissection Dilemma

The top two figures show how each of two shapes can be divided into four parts, all exactly alike. Your task is to divide the blank square into *five* parts, all identical in size and shape.







### 5. 100 Light Switches

I give you a row of 100 light switches, all in the off position.

Starting from the left, I ask you to flip every switch. Again starting from the left, I ask you to flip every other switch — so flip the 2nd, the 4th, etc. Again starting from the left, please flip every third switch. And so on: every fourth, then every fifth, etc, until on the last pass you flip only “every hundredth switch,” which means only the rightmost switch.

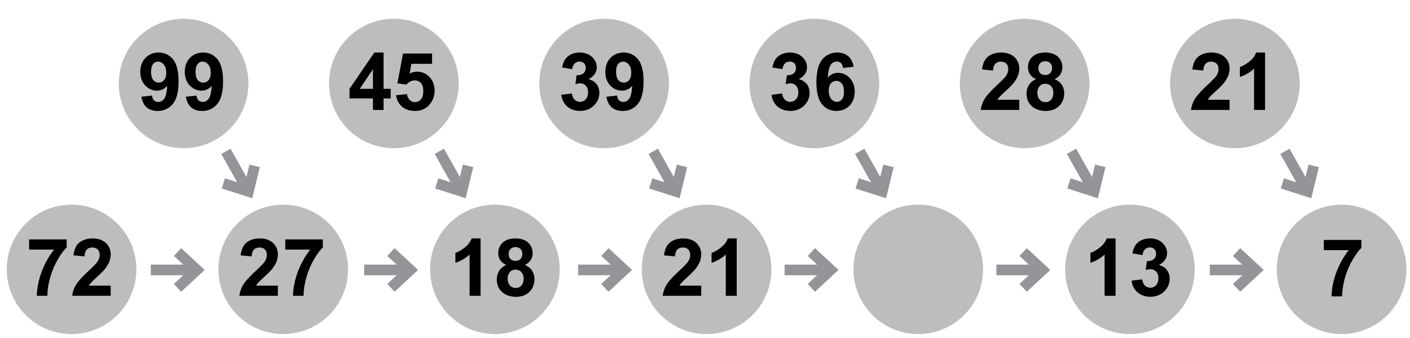
When we are finished, which light switches are in the on position, and which are in the off position?

### 6. George’s Ropes

George has six ropes. He chooses two of the twelve loose ends at random (possibly from the same rope), and ties them together, leaving ten loose ends. He again chooses two loose ends at random and joins them, and so on, until there are no loose ends. Find, with proof, the expected value of the number of loops George ends up with.

### 7. Are You Sure There’s No Typo?

Find the missing number in this sequence:

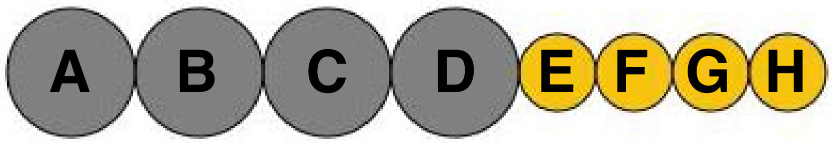


# SOLUTIONS

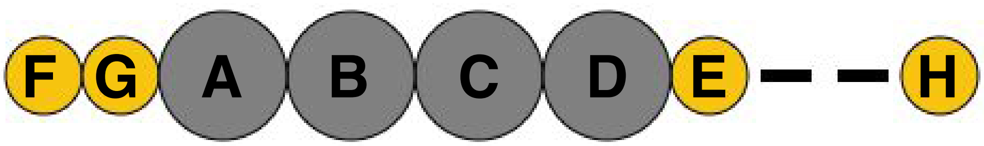
### 1. Coins

**Solution:**

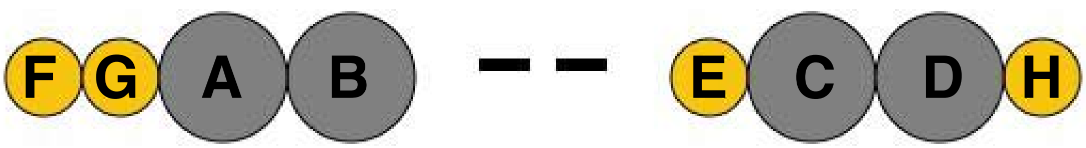
Here is your starting arrangement:



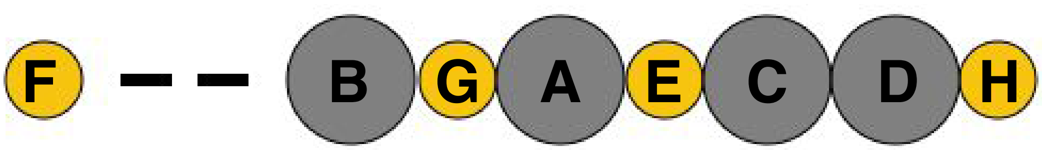

Move #1: Move coins F & G to the  beginning of the line.



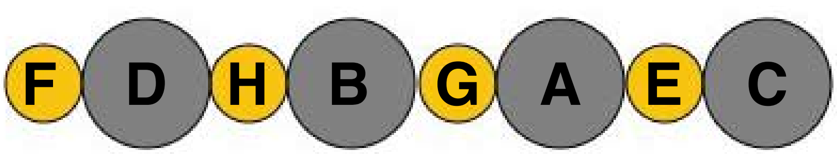

Move #2: Move coins C & D to the blank spots left behind by F & G.




Move #3: Move coins G & E to the blank spots left behind by C & D.




Move #4: Move coins D & H to the blank spots left behind by G & E.



## 2. Handshakes

**Solution:**

Because, obviously, no person shook hands with his or her partner, nobody shook hands with more than eight other people. And since nine people shook hands with different numbers of people, these numbers must be 0, 1, 2, 3, 4, 5, 6, 7, and 8.

The person who shook 8 hands only did not shake hands with his or her partner, and must therefore be married to the person who shook 0 hands.

The person who shook 7 hands, shook hands with all people who also shook hands with the person who shook 8 hands (so in total at least 2 handshakes per person), except for his or her partner. So this person must be married to the person who shook 1 hand.

The person who shook 6 hands, shook hands with all people who also shook hands with the persons who shook 8 and 7 hands (so in total at least 3 handshakes per person), except for his or her partner. So this person must be married to the person who shook 2 hands.

The person who shook 5 hands, shook hands with all people who also shook hands with the persons who shook 8, 7, and 6 hands (so in total at least 4 handshakes per person), except for his or her partner. So this person must be married to the person who shook 3 hands.
The only person left, is the one who shook 4 hands, and which must be my wife. My wife shook 4 hands.

## 3. Strawberry Ice Cream

**Solution:**

The children must be one 8-year-old and two 3-year-old twins.

First, consider all the trios of factors that equal 72 when multipied:

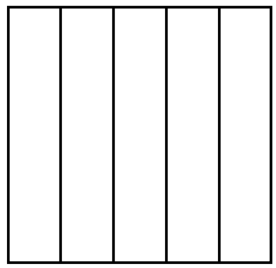
(72, 1, 1)  (36, 2, 1)  (24, 3, 1)  (18, 4, 1)  (18, 2, 2)  (12, 6, 1)
(12, 3, 2)  (9, 4, 2)  (9, 8, 1)  (8, 3, 3)  (6, 6, 2)  (6, 4, 3)

But as we know, we need more information. When the visitor declares that knowing the sum of the three ages is still not enough information, this gives us another clue. The only pair of permutations with the same sum are (8, 3, 3) and (6, 6, 2), which both add up to 14.

The final piece of information, that there is an *oldest* daughter, indicates the answer must be (8, 3, 3).

### 4. Dissection Dilemma

**Solution:**



### 5. 100 Light Switches

**Solution:**

All of the perfect squares (bulb #1, bulb #4, Bulb #9, etc.) will be on; the others will all be off.

A bulb gets switched (either on or off) any time a round of switching coincides with one of its factors. For example, bulb #12 gets switched on at round 1, off at 2, on at 3, off at 4, on again at 6, and then finally, off at 12. In fact, any bulb whose position number has an even number of factors will find itself in the OFF position at the final round of switching. And any bulb whose number has an odd number of factors will find itself in the ON position at the final round of switching.

What kind of numbers have an odd number of factors? Perfect squares.

### 6. George's Ropes

**Solution:**

The expected value of the number of loops he will end with is $\frac{6508}{3465}$

### 7. Are You Sure There's No Typo?

**Solution:**

This missing number is 12. Each number is the sum of all the digits from the two circles that point to it. For example, 9 + 9 + 7 + 2 = 27.