1. Red Hat, Green Hat

I brought my friends Miss Tall, Mister Medium, and Miss Short to the forest and arranged them in a line, with Tall in the back, Medium in the middle, and Short in the front, all facing forward, so that Tall could see Medium and Short, Medium could see Short, and Short could see nothing but the forest.

I blindfolded each, and then I reached into my bag of hats (which contained three red hats and two green hats, and nothing else), and put one hat onto each of their heads.

Removing their blindfolds, I assured myself that everyone could see the hats of the person(s) in front of him or her, but not his or her own.

I asked Miss Tall whether she knew the color of her own hat. She replied honestly that she did not.

I asked Mister Medium whether he knew the color of his own hat. He replied honestly that he did not.

But Miss Short immediately knew the color of her own hat.

What is the color of Miss Short's hat?

2. The Prisoners And The Light Switches

You and 99 other prisoners each live in solitary confinement in an island prison. No one ever sees or hears any other prisoners... until today. Today, the evil warden gathers you in a large hall you never even knew existed before, and tells you that he is being forced by budgetary constraints to close the prison. Not necessarily this week or even this year, but close it he must. And that means he has to decide what to do with you.

Being a sporting fellow, he has decided not to simply send you all off to the execution chamber, but rather to give you an opportunity to prove your cleverness and ability to work together... and to thereby earn your freedom.

After he is finished speaking, he'll give you all an hour to confer. Then you will all be escorted back to your soundproof, lightproof, vibration-proof cells. One by one, the warden's men will escort each of you into another secret room which contains nothing but two (standard, binary) light switches. The switches do nothing except to retain their position until flipped.

Once escorted to the room, the visiting prisoner will be obliged to flip one and only one switch. That is the entirety of the game.

At any time, any prisoner may alert the warden that he feels that all 100 prisoners have visited the secret room. If he is correct, all prisoners will be released. If he is wrong, all prisoners will be executed.

The warden, being evil, makes no guarantees about the order or even frequency with which he will assign prisoners to visit the secret room. For example, he might choose to have prisoners #1 through #98 each visit the room in turn, with #99 substituting in for every third prisoner, but with #100 not being sent until the 17th time through the roster. (You may take this paragraph to mean: “prisoners will be sent to the light-switch room in random order.”)

There is no opportunity for communication among prisoners except for the light switches, and for the hour that is about to commence.

Please describe the algorithm that will lead to your freedom.

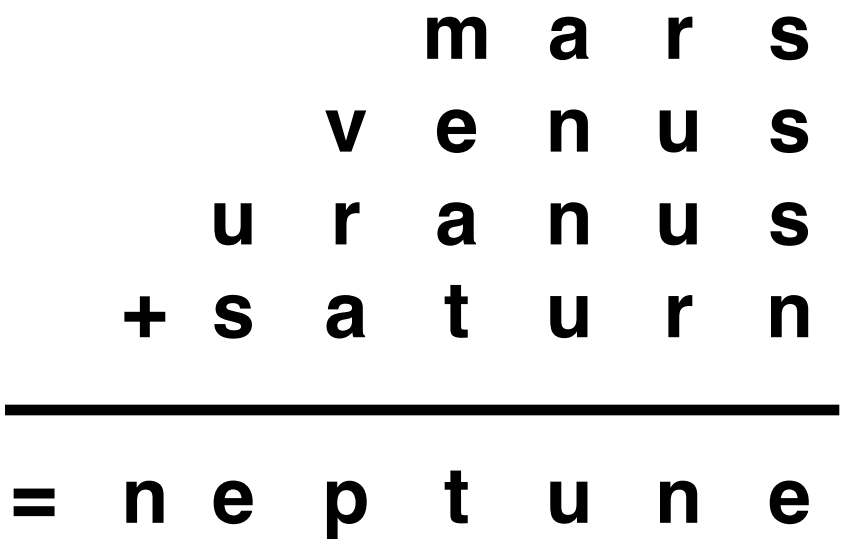
The full four points accrue to any team describing an algorithm that requires the switches to start in the DOWN position. A two-point bonus will be awarded for any algorithm that allows each switch to start in a random position.

3. Planets Addition

Each of the ten letters (m, a, r, s, v, e, n, u, t, and p) represents a unique number in the range 0 – 9. Likewise, each number is represented by only one unique letter.

The Question: What number does neptune represent?

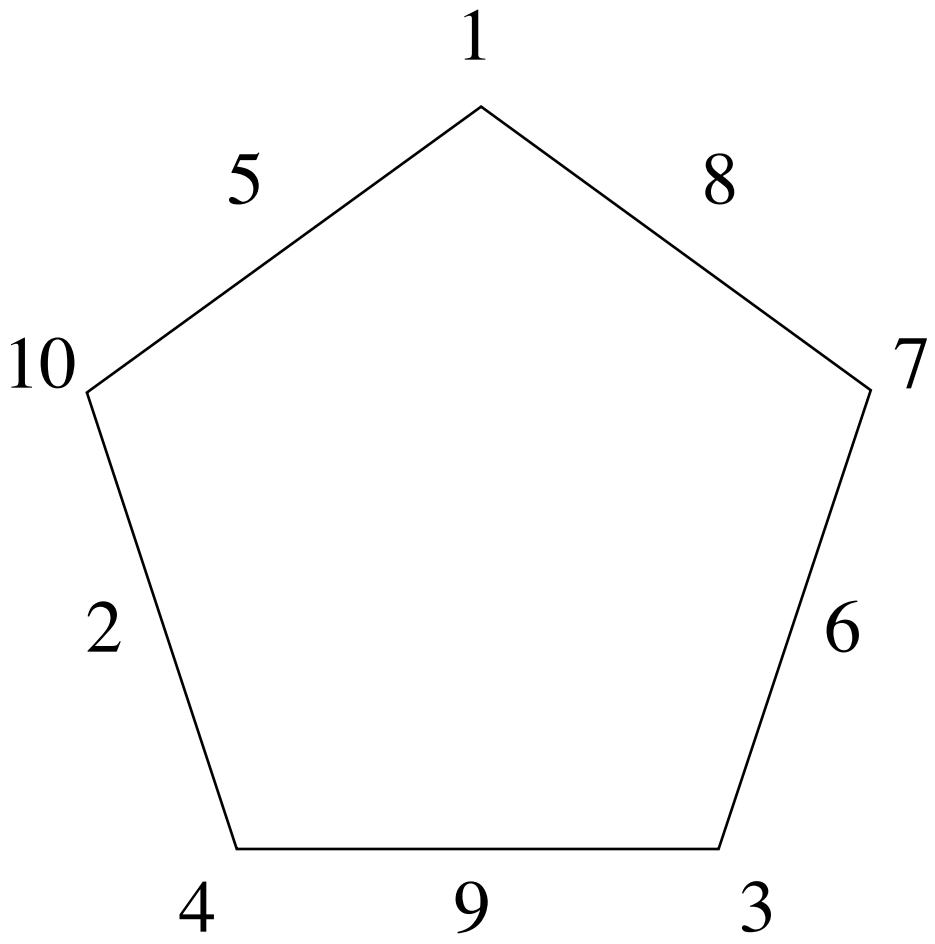
Hint: Numbers 1 and 6 are used most frequently.



4. Pentagon

The numbers 1 through 10 can be arranged along the vertices and sides of a pentagon so that the sum of the three numbers along each side is the same. The diagram below shows such an arrangement, with sum 16.

For four points: find, with proof, the smallest possible value for a sum, and give an example of an arrangement with that sum.



5. Saving the Guildarians

I, Prince Humperdinck, the evil Florinese despot, have successfully invaded the kingdom of Guilder. Shall I put them to death? Perhaps. It depends on the problem-solving acumen of the Guildarians! As leader of the Guildarians, your responsibility is to devise the strategy that will save the most of your people.

Here is my nefarious plan:

In the grand tradition of jailers everywhere, I will explain the game I am about to play, and give them some time to confer on a strategy.

Then, the game shall begin: I shall line them up in a room, tallest to shortest, back to front, so that each can see the long line of Guildarians in front of him or her, and nothing else. On pain of execution of all Guildarians instantly, I will swear each of them to utter silence and immobility.

Then I shall take my limitless collection of white and black hats, and put one on each head such that none can see the hat placed onto his or her own head, nor gain any information about what lies behind.

I shall ask each Guildarian in turn the color of his or her hat, starting from the back of the line and working forward. That Guildarian shall (under pain of execution as described above) reply either "white" or "black" and shall communicate **nothing** else in any way (using any means, secret or plain). Not to worry: my retinue of evil wizards can instantly detect any such foul play.

After each Guildarian has answered, I will lead him or her out of the room in such a way that no other Guildarian can see him or her at any time. No information can possibly pass between people in the room and people outside the room.

If that Guildarian's guess was correct, he or she shall go free. If not, he or she shall be sent to the fire swamp, where he or she will doubtless be eaten by Rodents Of Unusual Size.

As I said, you bear the responsibility as leader of the Guildarians to devise the strategy that will save the most Guildarians.

(However, as a professional courtesy, I will let you and you alone choose both the color of your hat and your position in line. You may do so after observing the hats of all your people, but only the people behind your chosen position in line will get to see your hat, just as though you were an ordinary citizen.)

What is the algorithm you'll use to save your people?

6. The Enigmatic Sequence

I invite you to consider the following sequence of numbers:

1, 3, 7, 13, 21, …

Then, please give me the following, for one point each:

(a) The 600th member of the series.

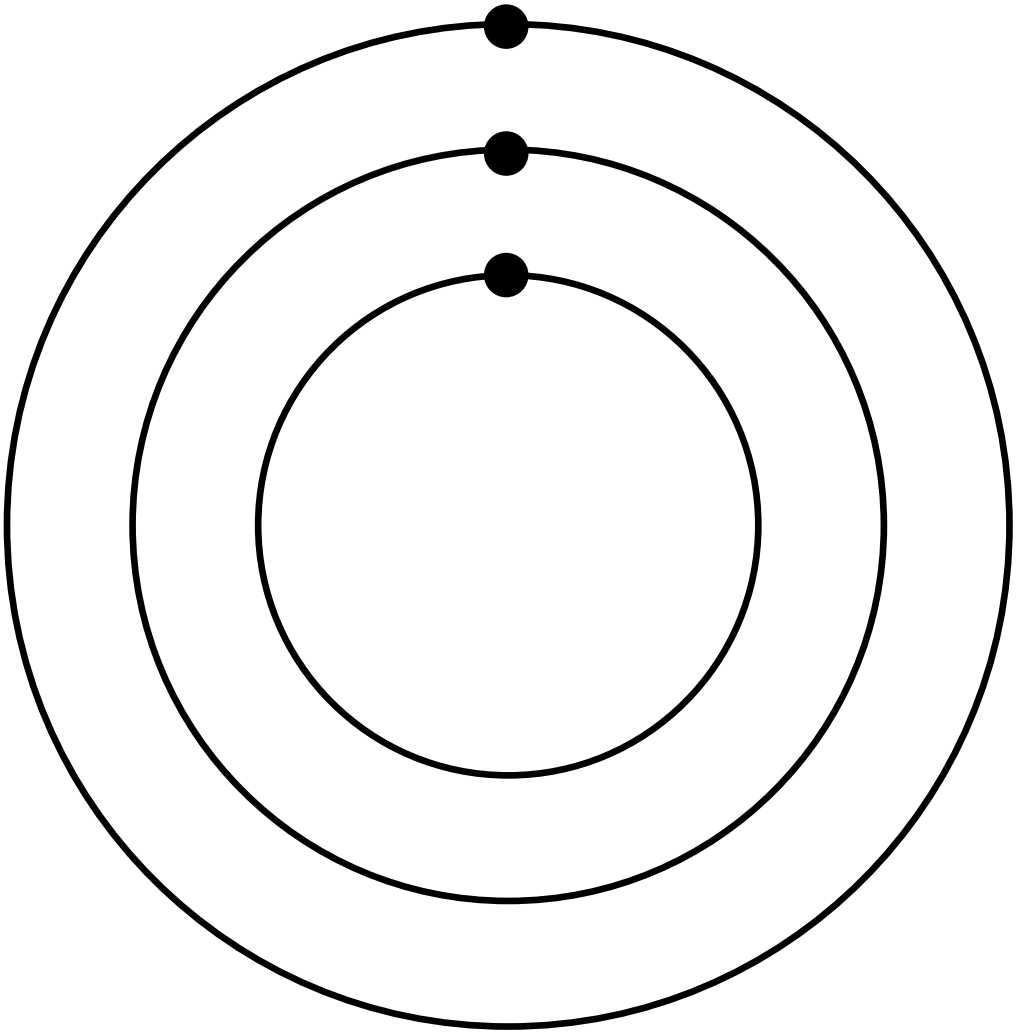
(b) A member of the sequence that has fewer than five digits, is a perfect cube, and is not the number "1".

(c) A member of the sequence that is a five-digit palindrome, and that can also be read as a binary number (by which I mean: whose digits are 1's and 0's only).

(d) The smaller of the two consecutive members that are exactly 1000 apart.

7. (TIE-BREAKER) Concentric Circles

The diameter of the outer circle is twice that of the inner circle. The other circle is midway between them. How many revolutions of the inner circle will there be before the three balls are again in line with each other. (Assume they're rolling in the same direction, at the same rate, starting at the same time.)



1. Red Hat, Green Hat

Solution:

My bag contained three red hats and two green hats.

So when Miss Tall told me she did not know the color of her own hat, we all learned that she was not looking at two green hats, for if she were, then she would know her own hat to be one of the remaining red ones. Therefore at least one of the frontmost two hats is red.

When Mister Medium then told me that he did not know the color of his own hat, we learned that he was not looking at a green hat, for if he were, he would know his own hat to be red, since at least one of the frontmost hats are known to be red.

Since Mister Medium was not looking at a green hat, he must have been looking at a red hat.

Hence Miss Short's hat is red.

2. The Prisoners And The Light Switches

Main solution (assumes both switches start down)

Assign one prisoner the role of Counter, and the other 99 the role of Drones.

On entering the switch room, each Drone is to move the right switch only, irrespective of its initial position, with one exception: the first time he sees the left switch in the up position, he is to move it down.

The Counter starts a mental tally at zero. On entering the switch room, she checks the position of the left switch. If it is down, she flips it up and increments her tally by one. If it is up, she leaves it up, and instead flips the right switch (irrespective of its position).

When her count reaches exactly 100, she alerts the warden.

Bonus solution (makes no assumptions about the starting positions of the switches)

As above, but with the following changes:

- Each drone flips the left switch down the first *two* times he sees it up.
- The counter alerts the warden when her count reaches exactly 198.

3. Planets Addition

Solution:

$$
\begin{array}{r}
4593 \\
20163 \\
695163 \\
+\ 358691 \\
\hline
=\ 1078610
\end{array}
$$

m = 4
a = 5
r = 9
s = 3
v = 2
e = 0
n = 1
u = 6
t = 8
p = 7

4. Pentagon

Solution:

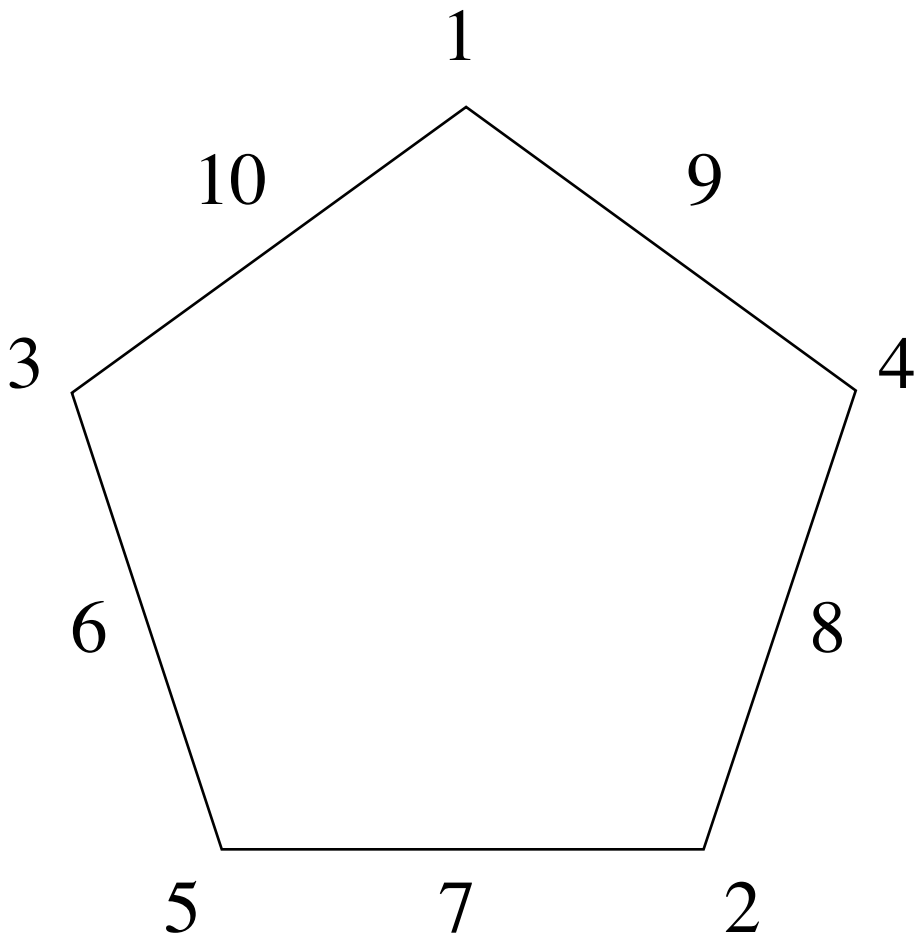
Your goal is to arrange the numbers around the pentagon such that the sum of the three numbers along each edge is the same, and also is as low as possible.

But notice that each corner number contributes to the sums of both the edges it touches, while each number in the center of an edge contributes to only to the sum of its own edge. In other words, corner numbers contribute twice as much to the sum as the center numbers do.

Therefore, to minimize the sum, you want the five smallest numbers in the corners, leaving the five largest numbers for the centers of edges.

Below is an arrangement of numbers that has this property.

The smallest sum is 14.



5. Saving The Guildarians

Solution:

The leader counts everyone else's black hats and if the total is even, he selects a black hat for himself. Then he takes his place in the back of the line.

Each person follows this rule on their turn: if the sum of the black hats visible in front of you plus the ones guessed behind you is even, guess black. Otherwise white.

Using this algorithm, everyone will survive.

Note: For Pi Day, I prepared a terribly clever solution for this puzzle that involved explaining the concept of parity, but it wasn't half as elegant as the one given above. Hats off to its author, the inimitable ***Seth Golub****.*

6. The Enigmatic Sequence

Solution:

*(Note that the **n**th member of this sequence is **n times [n-1], + 1**.)*

(a) The 600th member of the series is 600 * 599 + 1 = **359401**.

(b) A member of the sequence that has fewer than five digits, is a perfect cube, and is not the number "1" would be **343** (which is both 19 * 18 + 1 as also is the perfect cube 7 * 7 * 7).

(c) A member of the sequence that is a five-digit palindrome, and that can also be read as a binary number (by which I mean: whose digits are 1's and 0's only) would be **10101** (101 * 100 + 1)

(d) The smaller of the two consecutive members that are exactly 1000 apart is **249501** (500 * 499 + 1, which is exactly 1000 less than 250501, which in turn is equal to 501 * 500 + 1).

7. (TIE-BREAKER) Concentric Circles

Solution:

There will be six revolutions of the inner circle before the three balls are again in line with each other.