Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

1. CHOCOLATE BARS

Consider a chocolate bar that's a 3x6 grid of yummy squares. One of the squares in the corner of the bar has an X on it. With this chocolate bar, two people can play a game called "Eat The Rest." Here's how to play:

One person starts with the bar and breaks it into two pieces along one horizontal or vertical score line. The person hands the piece containing the X to the other person and sets the other piece aside. The second person then repeats this process (breaking the bar into two pieces, handing the one containing the X back to the first person, and setting the other piece aside). The game continues until someone — the loser! — gets handed the single piece with the X.

The person who ends up with the X only gets to eat that piece, but the other person — the winner — gets to Eat The Rest.

THE PROBLEM: Is there a strategy that ensures victory no matter what your opponent does? If so, explain it. You may assume that you get to choose whether you play first or second, if your strategy requires it. If no such strategy exists, briefly show why not.

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

2. THE PUZZLING PAINTERS

Juliana and Wes have given up curating puzzles and running puzzle events and have decided to take up painting houses.

They've gotten into a great rhythm. They each paint at a constant rate (Juliana, for example, paints at the rate of 600 square feet per workday), and they each have exactly the same workday (which includes a lunch break in the middle).

This is a problem about a particular wall that they like to practice on. It turns out that Wes can paint it exactly twice in one work day.

Yesterday, Wes started painting this very wall, working outwards from one corner. Two hours before lunch, Juliana, fresh from finishing some other painting job, arrived and started in a different corner. Together they finished the first coat. At that moment, Juliana got a call to do some other job, so she left, but Wes stayed to finish the job. Wes went immediately back to his starting corner, and started the second coat.

Wes put his second coat exclusively on the parts he himself had painted earlier in the day, stopping as usual for lunch and resuming afterwards. There came a moment at which Wes realized he had finished the second coat on his part of the wall, and was about to begin the second coat on the part that Juliana had painted. At that moment, Wes happened to look down at his watch and noticed that it was an hour before quitting time.

THE PROBLEM: How big is the wall?

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

3. ALICE, BOB & CHUCK

For you hat-puzzle aficionados, the hats are back, and they're all black! And this time, with numbers!

Here we go:

Alice, Bob, and Chuck are sitting in a circle. Each is wearing a black hat with a positive integer written on it. Each person can see the other two numbers but cannot see their own. They are told that one of the numbers is the sum of the other two.

Alice says, "I cannot determine my number."

Bob then says, "Given that, I still cannot determine my number."

Chuck then says, "Given those two statements, I still cannot determine my number."

Alice then says, "My number is 50."

THE PROBLEM: Assuming they have all used valid reasoning, determine Bob's and Chuck's numbers.

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

4. THE TRAINEE TECHNICIAN

A superconducting cable, containing 120 superconducting wires, has been laid deep underground between two telephone exchanges located far, far apart.

Unfortunately, after the cable was laid it was discovered that the individual wires are all white and unlabeled. There is no visual way of knowing which wire is which, and thus the cable can't (yet) be used.

You are a trainee technician and your boss has offered you a raise if you would please identify and label all the wires, at both ends, without ripping it all up. The idea is that you can write whatever you like on each of the 240 ends of wires, as long as, by the end of your task, you know which wires are which.

You live at one end of the cable, and you have the following items: a battery and a light bulb (to test continuity), labels and a marker (for labeling the wires), and *one* round-trip ticket to the city at the other end of the cable.

THE PROBLEM: Please explain how you will get your raise.

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

5. PARTITION A MILLION

THE PROBLEM: Please write one million as the sum of a prime and a perfect square.

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

6. RED AND GREEN DICE

I have two fair dice: one red and one green.

I roll the red one and I write down the number it shows Then I roll the green one and add that number and write down the result. So, for example, if I rolled Red 5 and Green 3, I have written down “5, 8”.

Then, leaving both dice on the table, do the add-and-write thing again. And again, and again, continuing forever.

So, in the above example, I’d have 5, 8, 11, 14, and so on.

If I had rolled 1 and 1, I’d have 1, 2, 3, 4, 5, 6, ...

If I had rolled 3 and 2, I’d have 3, 5, 7, 9, 11, ...

THE PROBLEM: Assuming a fair (random) roll, what’s the chance that my list contains a perfect square?

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

7. CANNIBAL CROSSING

Three cannibals and three anthropologists have to cross a river.

The boat they have is only big enough for two people. The cannibals will do as requested, even if they are on the other side of the river, with one exception: if at any point in time there are more cannibals on one side of the river than anthropologists, the cannibals will eat them.

(Note: One anthropologist can not control two cannibals on land, nor can one anthropologist on land control two cannibals on the boat if they are all on the same side of the river. This means an anthropologist will not survive being rowed across the river by a cannibal if there is one cannibal on the other side.)

THE PROBLEM: What plan can the anthropologists use for getting everyone across the river alive?

Please diagram your strategy on the back of the scoresheet, using X to stand for cannibals, O to stand for anthropologists.

Ask a Scientist Pi Day Puzzle Party 3.14 2014

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

8. [{n}] = {[n]}

Suppose it is known that

{3}=47
{10}=138
{1}=39
(and so forth)

and that

[1]=5,
[20]=43,
[99]=201
(and so forth)

THE PROBLEM: Please find the one prime number for which [{n}] = {[n]}.

SOLUTIONS

→

1. CHOCOLATE BARS

Only the person who goes first can ensure victory. Break it in half (sticking the other sucker with a 3x3 section), and the opponent can't win.

If opponent then gives you a single row or column, you hand him the X. If opponent gives you a 2x3 (his only other option) you hand back a 2x2, Next turn he must give me a single row or column.

2. THE PUZZLING PAINTERS

The area of the wall is 900 sq ft.

Since there are two coats of paint, the region Juliana painted in the morning could have been painted twice by Wes in one hour Thus, Wes takes 30 minutes to paint this region once. Working alone, Wes can finish one coat on the wall by lunchtime; so, 30 minutes before lunch, Wes could have painted everything except what Juliana painted. Thus, Juliana took (2 hours minus 30 minutes) = 90 minutes to paint what Wes can do in 30 minutes. Since Juliana paints 600 sq ft per day, Wes can paint 1800 sq ft per day, and the area of the wall is 900 sq ft.

3. ALICE, BOB & CHUCK

Bob and Chuck's numbers are 20 and 30.

In short, the whole thing rests on the idea that the fourth speaker (a.k.a. Alice) knows her number. In other words, other numbers might allow the third or fifth speaker to be the first to have an answer, but only 20 and 30 fits the bill for Speaker #4.

1 and 49, and 2 and 48 (etc) all suffer from the problem that Alice still can't determine her number when it comes around to her.

Example:
a=50, b=2, c=48:
a doesn't know whether her number is 46 or 50;
b doesn't know whether 2 or 98;
c doesn't know 48 or 52;
a still doesn't know.

Whereas:
a=50, b=20, c=30:
a doesn't know 10 or 50;
b doesn't know 20 or 80;
c doesn't know 30 or 70;

...but now A knows that she isn't 10, because if she were, c's options would have been 0 and 30, and since 0 is not possible, c would have known his number. Therefore Alice knows her number is 50 because it cannot be 10.

Let's try to run that same logic on the first example:
a=50, b=2, c=48:
a doesn't know whether her number is 46 or 50;
b doesn't know whether 2 or 98;
c doesn't know 48 or 52;

..."but now A knows that she isn't 46, because if she were, c's options would have been 44 and 48, and since 44 is not possible…" Oh, snap! Turns out 44 *is* possible, and we're still nowhere.

4. THE TRAINEE TECHNICIAN

At one end label a wire "A". Then join two wire and label them both "B', then tie three (not already joined) wires together and call them each "C"....continue until all the wires are joined together in groups of 1, 2, 3, 4, 5, etc....for a 120 strand cable. NOTES that the largest group will have 15 wires.

Now walk to the other end.

Using a (battery and light bulb) it is now possible, for example, to find the wire that wasn't joined to any of the others. It is similarly possible to find which wires are in a pair, which is joined in a group of 3, etc. Each time a group is found the technician should label it with the letter for the group, so the single wire is labeled 'a', the pair are each labeled "A", etc....this now matches the other end.....the letters will go up to "O". Now take "A", "B", up to "O" and join them together in a group and label each one with "15", so we have cable "A15", "B15', "C15", up to "O15". Take the second and last "B'" wire and join it with a remaining "C", "D", up to "O" and label these each "14' so we have "B14", "C14", up to "O14". Repeat this until at the end there will be a single "O" cabled labeled "O1".

Now walk to the other end.

Now untie all the old connections and identify the group labeled "1", "2", "3" ..."15" at which point each wire at each end has a unique classification.

5. PARTITION A MILLION

$998{,}001 + 1999 = 1{,}000{,}000$

Let t (for Thousand) be 1,000. Then one million is t^2.

Let s (for Square) be the number for which s^2 is our perfect square.

Let p (for Prime) be our prime. Then $t^2 = s^2 + p$. Which means $t^2 - s^2 = p$.

But, crucially, $t^2 - s^2 = (t+s)(t-s)$ by the rules of algebra.

So $(t+s)(t-s) = p$. Recall that $t=1{,}000$ and s is an integer...
...and, also crucially, a prime is always p times 1, with no other factorization possible.

So $t-s=1$ and $t+s$ is the prime. Since $t-s$ is 1 and $t=1000$, s must be 999.

And since $t+s$ is the prime, $p = 1999$.

And since the square plus the prime equal a million,the square must be $1{,}000{,}000 - 1999$.

Which is 998,001. Which is indeed a perfect square: it's 999^2.

6. RED AND GREEN DICE

Total: $27/36 = 3/4 = 75\%$

Note that perfect squares end in 0, 1, 4, 5, 6, or 9; never 2, 3, 7, or 8.
(i.e. they are all congruent to 0, 1, 4, 5, 6, or 9 (mod 10)).

There are 36 possible rolls.

Of the 6 with $g=1$, all contain the perfect square 9.

Of the 6 with $g=2$, all contain either 9 or 16.

Of the 6 with $g=3$, $r=1$ and $r=4$ yield 16; $r=3$ and $r=6$ yield 9; $r=2$ and $r=5$ yield no squares.
This is because all perfect squares are congruent to 0 or 1 (mod 3).

Of the 6 with $g=4$: all squares are 0 or 1 (mod 4), so $r=1,4,5$ yield squares, and $r=2,3,6$ do not.

Of the 6 with $g=5$: all squares are 0,1,4 (mod 5) so $r=1,4,5,6$ work; $r=2,3$ do not.

Of the 6 with $g=6$: all squares are 0,1,3,4 (mod 6) so $r=1,3,4,6$ work and $r=2,5$ do not.

So the red/green die rolls that will never have a perfect square in their infinite list of sums are:
(2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (5,3) (5,6) (6,4)

That leaves 27/36 rolls that will have perfect squares in their lists.

7. CANNIBAL CROSSING

X = cannibal, O = anthro, <> shows direction of boat)

Step 1)	XXXOOO	[everyone starts together on left bank]
Step 2)	XXOO > XO	[one X and one O cross together to the right bank]
Step 3)	XXOOO < X	[one O returns alone to the left bank]
Step 4)	OOO > XXX	[... and so on]
Step 5)	OOOX < XX	
Step 6)	XO > XXOO	
Step 7)	XXOO < XO	
Step 8)	XX > XOOO	
Step 9)	XXX < OOO	
Step 10)	X > XXOOO	
Step 11)	XO < XXOO	
Step 12)	> XXXOOO	

8. [{N}] = {[N]}

2 is the prime solution. (-8 is also a solution.)

$[n] = 2n+3$

$\{n\} = n^2 + 38$

$\{[n]\} = (2n+3)^2 + 38 = 4n^2 + 12n + 47$

$[\{n\}] = 2n^2 + 79$

$\{[n]\} - [\{n\}] = 2n^2 + 12n - 32 = 2(n+8)(n-2)$,
which equals zero when n is the solution

Thus $n=2$ (or -8).