

TRANSCRIPT: ASK A SCIENTIST

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(Slightly edited for clarity)

Speaker: Dr. Keith Devlin — Executive Director of Stanford's Center for the Study of Language and Information; author of *The Math Instinct*

Topic: The Math Instinct

Host: Juliana Gallin

Juliana Gallin: Hi, thanks for waiting and thanks for coming to Ask a Scientist. Tonight we'll be speaking with Keith Devlin, he's NPR's "Math Guy"—when are you on exactly, Saturday mornings?

Keith Devlin: Yes, before Car Talk. Every so often.

Gallin: Okay well, tune in and find Keith sometimes on Saturday mornings before Car Talk. He's also a consulting professor of mathematics at Stanford as well as the author of these three books you see here: *The Math Instinct*, *The Math Gene*, and *The Millennium Problems*—and I know there are many more, correct?

Devlin: Yes, twenty-four.

Gallin: Wow! I didn't realize there were *that* many! Well tonight we're going to be talking about this one, *The Math Instinct*. So let's all welcome Keith then, and hear what he has to tell us. *[Applause]*

Devlin: Thank you—I didn't quite expect there to be as many people as this! You guys don't have televisions or something? *[Laughter]* I know for sure you don't have parking spaces in the streets around here—but it was nice walking around San Francisco, having found a place to park and walking a couple of hours back to the restaurant. *[Laughter]*

Okay, so yes I'm going to be talking about this book, *The Math Instinct*. The title's not mine, the publisher thought Stephen Pinker's book *The Language Instinct* was worth ripping off. And the subtitle is: Why you're a mathematical genius—along with lobsters, birds, cats, and dogs. The book is ostensibly about cool things that animals can do and cool things that people can do without being consciously aware of it, and some reflection upon that. But actually that's not what it's really about. It's really about *what is mathematics* and what does it mean to describe some kind of thinking, some kind of cognitive activity, as mathematical thinking. I think that needs to be reflected on for a variety of reasons, not least because people often make claims about, you know, no child left behind, everyone should be able to do math—whatever that means—by a certain age. Or to achieve proficiency in mathematics, as if mathematics was one thing rather than a whole spectrum of cognitive abilities—different people having different amounts—and if you have a lot of those abilities you can call yourself a professional mathematician. But one of the things I try to describe in the book is that if you sort of parse mathematical thinking out into all of the things that constitute mathematical thinking you'll find it all over the place, it's not limited to humans by any means. And in fact there are instances of what we would call mathematical thinking that different creatures have to a quite sophisticated degree.

So let me begin by describing, or recounting, one or two of the stories that I tell in the book. And the book is a collection of stories—actually what I do is I try to tell a bunch of little stories about animals

and people, and then try to let the general theme just sort of wash out of that without sort of banging it on the head with a mallet.

My favorite animal trick, or animal capacity, if you like, is exhibited by a tiny little creature called a Tunisian desert ant, which is an ant—*Cataglyphis fortis* is its proper name. As you might imagine, if it's called a Tunisian desert ant, where does it live? Tunisia, yep. It lives in the sandy desert, and more precisely it lives underground in the desert. They build their nests underground, and as everyone knows they have sort of a strong societal structure where they all have different roles to play. And some of the ants, their job is to go out and forage and find food. So the food gatherers, they'll come up out of the nest—they've got this little tunnel and there's a little hole in the top, in the sand, about the size of your small fingernail. And out pops this little ant, and it will wander across the desert—and this is the featureless sandy desert, *Lawrence of Arabia* stuff, for those of us who are old enough to remember *Lawrence of Arabia*. [Laughter] It comes out onto the sandy desert and wanders around looking for food. It'll go left, and it'll go right, it can wander as much as 100 meters. If you look at these things they really do meander around. By the time it finds the food it could be 100, 110 meters or so away from the nest. And then it'll find a dead insect or a leaf—food is scarce in the desert, but it eventually finds something.

Then the interesting thing happens. This creature has wandered around—and by the way, scientists have gone out and they've stuck little tracking devices on these things, and they track them! They really do follow quite remarkably complicated looking paths, left, right, and so forth. But the interesting thing is once this little critter finds its food it turns around and faces where the nest is and travels the exact distance it needs to find the nest, looks around the last few centimeters or so, finds the hole, and pops down with the food. We know that it's really doing that because the scientists went along, followed the ants while they were looking for food, and when an ant found the food, the scientists would pick it up and move it somewhere else, some distance away. And the ant would follow the direction it *should* have followed to get to the nest. It would go the exact distance it *should* have gone to get to the next. And then of course there's no nest, so it just wanders around bewildered because it thinks the nest should be there. [Surprised murmurs]

How on earth is it doing that? Contrary to what lots of guys claim, those of us when we drive cars and we say we can always find our way back, it's not true. [Laughter] Imagine if you were doing this. You're out in the forest somewhere or in the desert, you don't have a compass, you don't have a map or anything. All you've got is the ability to remember you turned left here and then you turned right 23.5 degrees here, then you went another 15 feet and turned left 17 degrees there, then you turned 19 degrees and went another 4 feet... You could do it if you kept track of exactly how far you've traveled in each direction, and you measure exactly—and it has to be pretty close to exact, because after you've wandered around a bit those angles and distances will all add up together, they'll all accumulate, and it actually is quite sophisticated trigonometry to figure out the exact angle and exact distance you need to go back. Humans can do this. The ancient mariners used to do this, it's how they found their way across the oceans. During the Apollo missions to the moon this was the back-up system that the astronauts did in order to make sure that if anything went wrong they would find their way back. And they used it on the Apollo 13 when they had to abort the mission and come back—it wasn't really Tom Hanks but he played it in the movie [laughter]—they found their way back using that [method].

So, humans know how to do it but it involves using accurate time keeping, and accurate speed measurement so you can measure the distances accurately, and accurate measurements of angles. You have to be very precise because if you get off slightly in the distance or the angles, when you do this process of accumulation—sometimes called performing a *path integral*—then you'll be slightly off in the distance or the direction. So humans can do it using equipment and trigonometry, but these little

critters just do it. They're not using sense, they're not using any other kind of directional things, because as I said, if you pick them up and move them the one thing they remember is the direction and the distance. If you look at this and apply Occam's Razor and say, what's the simplest explanation? you actually come down to just one explanation: *in their own terms* these little creatures are doing trigonometry and measuring distances and angles very, very precisely.

There we have the case of a creature, a seemingly lowly creature, that can do something that when humans do it—I'm not saying that the creature does mathematics in the sense that we're doing it, not at all. In fact let's just assume that the critter is not thinking consciously at all, may not have anything that we recognize as consciousness, probably is not aware of its own existence, doesn't have any kind of language, however it has an instinct. It does certain things. It has an instinctive behavior. This is why the book's title is something I could live with. Because this little creature is simply following an instinct. It's when we try to interpret it in human terms that we say, well, in terms of what we would do, it's doing trigonometry, it's doing *dead reckoning*—that was the term used by the mariners, and then the more recent use of the term came in the 20th century, in early aircraft days when pilots were finding their way around before GPS and those things came along.

So the book has a whole range of examples like that, of critters that can do things. The example with the desert ant is sort of the knock-down one, because you really can't argue your way out of that one. As you look at what animals do, for some you can say, well, it's *sort of* mathematical but you may not want to describe it as mathematical. And you do have to be careful because if you go around looking at creatures, you know, all creatures do something remarkable. That's how natural selection works, anywhere in the world except Kansas. *[Laughter]* So the fact that they've survived means there's got to be one thing they do which, in human terms, is pretty smart. All creatures have very sophisticated things that they do in their own world, things that we look at that seem very sophisticated. And sometimes you can look at them and say that in human terms, that's mathematics. And other times you can't. An example that I give in the book, an example that's sort of on the borderline, is beavers building dams. If you look at a beaver dam, it sort of curves in a nice arc facing the flow of the water. And you think, well, that's the way humans build dams—Hetch Hetchy or Hoover Dam—it's in a nice curved arc that faces the water. And you're sort of inclined to think that maybe beavers are wonderful engineers that know all about pressures and so forth. But then you think, wait a minute, all it would take—you have to apply Occam's Razor, you have to say if beavers are building dams, what's the least that they would have to do? And when you think about it, all the beaver would need to do to build a dam is have an instinct to stick twigs in a slow moving river. Because if they start sticking twigs and branches in a slow moving river, the very fact that the water is moving in a certain direction will push those twigs together and that shape will emerge. The reason humans build them that way is because that gives you the equilibrium with the water, but the water and the twigs together will generate that equilibrium. And so yes, there is mathematics being done in the beaver dam, but it's not really being done in the mind or the body of the beaver. If it's being done at all it's being done by the beaver and the river considered as a system altogether. In fact if anything is *doing* the mathematics, it's the river. So you've got to be a little bit circumspect in playing the game that I play in the book and looking for things that animals do and identifying them as mathematics.

At the far end of the spectrum, or the silly end of the spectrum, if you went to Coit Tower and stepped off the top, now, in a sense, you've solved Newton's equations of motion. *[Laughter]* I wouldn't recommend doing that—it's what we in the education business call *high stakes testing*. *[Laughter]* So yes, there's mathematics being done in a sense, but it's being done by the universe—which is a silly way of saying what we would normally say as, “mathematics describes what the universe does.”

So there is a whole spectrum of different things that creatures do and how you look at them. However, as I try to make clear in the book, there's a bunch of things that creatures do that seem to me to be inescapably mathematics. That is, what you *can* say is that this creature, like the desert ant, when it does its instinctive thing, we, as humans, are forced to interpret that in terms of mathematics. We can't make sense of it except by saying it's doing mathematics. And there are a bunch of examples like that.

Now, I'm sort of going a bit further than that and I'm saying if we can't describe it in any meaningful way *other* than to say it's doing mathematics—doing something *we* do as mathematics—I'm saying, well, let's just say it's doing mathematics! At that point people might say, no, you've gone one step too far Devlin, you've ascribed mathematics to that animal and it's very unreasonable. Well, that's a point of view. But if you want to adopt that point of view, you need to be consistent. You'd better not say that that little calculator in your pocket does arithmetic, or does multiplication, or does addition, or does anything mathematical or arithmetical. You better not say that that graphing calculator draws a graph. You better not say that your desktop computer is solving an equation, or doing a derivative, or performing an integral. And we use those terms all the time. We have machines that we've built—starting in the 19th century with mechanical devices, and then electrical devices, and now electronic devices—that we describe all the time as if they're doing mathematics. Now that's reasonable—if you take a graphing calculator, it will pass most high school exams in mathematics! The *calculator* will pass the exam. The student won't pass the exam, and that's why teachers get worried about these things, because it's clear that mathematics is being done. You don't need to know mathematics, you just need to know how to use the machine. Software systems like Mathematica, they will pass most first-year and many second-year college math exams—unless the professor is very careful to develop an exam that can't be done that way. But a traditional freshman or sophomore level university exam, something like Mathematica will be able to solve it. And in fact, these days when engineers design buildings and spacecraft and all kinds of things, the engineers don't spend time doing the math. They don't need to, they simply let the system do the math.

So it's part of our culture to describe machines as “doing mathematics.” And most people would think it was unreasonable for someone to say, no, the machine's not doing mathematics, it's just doing something else. And yet that's really what's going on, right? That calculator, that computer, is an electronic device. There's no brain in there. There's no consciousness in there. There's nothing like what we would call cognitive activity. All there are are electrons flying around, close to the speed of light, flying around a dynamically changing circuit. And yet we happily describe that as mathematics. *It's* not doing anything—when you press the *on* switch, from then on, all that machine is doing is following the laws of physics, the laws that govern the way electrons flow. That's all it does. It does it in a complicated pattern, but that's all it's doing. And yet we interpret that behavior and say it's doing mathematics, it's doing an integral, it's doing a derivative, it's adding numbers, it's drawing a graph—that's the way we interpret its activity. So we say that certain machines do their thing, follow their instincts, if you like, because it's an instinctive behavior—if you want to anthropomorphize what the computer or calculator are doing, perhaps a more accurate way is to say that it was just following its instinct, doing what's natural, what it was built to do.

Now, why do we interpret it as mathematical? Well, first of all because we want to make sense of the world, and we make sense of that activity in terms of calling it mathematics. It works because we designed that machine to make it work, obviously. But the reason the guys who design these machines get rich is because it's actually not easy to design things to do that, right? It takes a lot of skill to design a machine that, when it does its instinctive thing, humans can interpret it as a certain activity, be it word processing or whatever. We interpret *its* thing in *our* language. And as I said, it makes sense, because we designed it that way. But wait a minute! Natural selection “designs”—in quotation marks. Because of the way natural selection works, it optimizes. And optimization produces local optimal solutions. And

those are invariably mathematical solutions because that's what an awful lot of mathematics is about, it's about optimization. So the desert ant has evolved to do its instinctive thing, and its instinctive thing is being always able to know where home is and how to get there. That's its one great trick. And the only way we have, I think, of describing its great trick is to say it's solving one particular mathematical problem.

And if you look at the other examples in the book, I talk about creatures doing things and what they do is they've each got their one mathematical problem that they solve very, very, very well. But it *is* a matter of interpretation. In other words when we say that someone or something is doing mathematics, it's very much like interpreting a Picasso or anything else, we're interpreting something in the world, making sense of it in our terms. Where is the mathematics being done? Well, if anywhere it's in the eyes of the beholder. We look at a calculator and in our eyes it's doing mathematics. We look at a desert ant and in our eyes it's doing certain kinds of mathematics. And there are other creatures that are doing things that are borderline mathematics. It's not always reasonable to call it mathematics—sometimes you just say, well, it's sort of vaguely mathematical.

So that's what a lot of the book is about. It's about recognizing that there are lots of different kinds of thinking that we classify as mathematics. And many of those things can be found all over in the animal kingdom, and when you look at the world that way what you're doing is you're interpreting natural behavior—or engineered behavior in the case of computers and calculators—you're interpreting it in human terms. That's the first part of the book, that's what the story is. And then of course the issue is, well, okay, what about humans? What instinctive capacities do we have? We don't have this ability to find our way back accurately after we've got lost. It only takes a couple of diagonal streets, and San Francisco has a few of them, to just throw you right off of where you are. *[Laughter]* And by the way, there's a whole chapter on things like migration. When you get into migration of monarch butterflies and whales and salmon and so forth, it's just mind blowing. I mean the monarch butterfly, the one that's born up in Canada because its great-great-grandparent came from Mexico at the end of the last season—that creature that's born there knows how to find its way right across North America, down the Pacific Coast, stopping off in Santa Cruz, then stopping off in Santa Barbara. Really neat places—they're not stupid are they! *[Laughter]* They go all the way down to this little thirty-acre pine forest west of Mexico City where they spend the winter before another generation makes its way back again. It's hard not to think of that as being not just mathematics, but mathematics that's built in through a blueprint, because the creatures that are doing it are not the ones that did the original journey. So in a sense the blueprint determines the instinctive behavior, which we interpret as doing trigonometry, if you like, just like when the engineer designs a computer chip [inaudible].

So what kinds of things do humans do that have this capacity that we might interpret as mathematics? Well, first of all, there's two things to be said. It may be that if you go back many generations, into our ancestry, maybe pre-Homo sapiens, our ancestors may have had all sorts of capacities. But we did something different, we developed this huge neocortex, we became consciously reflective. And we were able to do something that seems pretty unique in the animal kingdom, we've developed the capacity of being able to learn new tricks. We seem to have an unbounded capacity to learn new tricks, new kinds of things, and to explicitly develop mathematical methods that allow us to do things that creatures do—to develop aircrafts that help us to fly, submarines that help us to go under the water. We have found a way to do things that different creatures have been doing all this time, and that seems to be a rather unique sort of trick, this ability to sort of step outside of our existence, reflect on it, plan, design, build things, do mathematics and so forth. So we have cognitive tools, and with the cognitive tools we develop physical tools. We've sort of taken a different evolutionary path, relatively recently, and this allows us to do various things. And about 10,000 years ago we embarked upon what we now call mathematics, around 10,000 years ago in Sumeria when society reached a sufficient level of complexity that people

began to want to trade property, and to keep track of how much each person owned, and have some kind of tokens for measuring exchanges. They invented, or discovered—whichever way you want to describe it, and it probably felt like discovery but I think it was really invention—numbers. Whole numbers— one, two, three, four—to the best of our knowledge were developed in Sumeria about 8,000 BC. And then that began this path—and 10,000 years is a *very* short time—within the last 10,000 years we’ve developed this other stuff, very new stuff, Johnny-come-lately stuff that we call mathematics. And it’s just like the last person to move into the neighborhood, the first thing they do is say, “We don’t want any more houses built here. There’s enough people, thank you very much.” We’ve sort of developed this new way, and we tend to think that that is the only way of doing mathematics—because it’s new to us. Most of it’s incredibly new. Calculus is only 350 years old. Most of the known mathematics was developed in my own lifetime, and I’m not *that* old. [Laughter] It’s been developed relatively recently. Most mathematics has been developed within the last 50 or 60 years, so this is recent stuff. It’s a new trick that we’ve learned. And like anyone that’s learned a new trick, we tend to think that’s the only way to do it. But actually, if you stop and look, nature has been doing this trick for years in different ways. The way *we* can do it, what’s unique about that, is it’s abstract, it’s generic, it’s universal. We can take our mathematics and apply it to all kinds of domains—engineering, medicine, communications, technologies, we can apply it to study society, money supply and economics and so forth, management science. We can apply this mathematics all over, and so we’ve got sort of a universal, one size fits all [inaudible]. And that’s new and it’s novel and we’re justifiably, as a society, proud of it—well some of us are [laughter]—but let’s not overlook the fact that in some sense it’s not new, we’ve just got a new way of doing it.

The thought I was going to lead into was, what about the things that humans do that are really pretty smart? I’ll pick one example because it’s the one that you probably thought I was going to talk about anyway, and that’s just numbers. They were the first thing that we had [inaudible] mathematics, and they are almost unique to human beings. Some creatures have *some* numerical capacities. We, as a species, all the evidence suggests, are born not just with a capacity to deal with numbers, but some of that capacity’s already implemented.

Some remarkable research came out of MIT in 1972—no, 1992—no, it was 1972. You reach a certain age and time just sort of concertinas. [Laughter] ’72, ’82, ’92, you know the Beatles had already split up so it’s all the same era. After the Beatles split up and Bob Dylan had his motorbike accident, to me, everything since then has been five minutes. [Laughter] So, Karen Wynn, 1972, at MIT, she showed that young children, five or six months after birth, knew that one plus one is two, one plus two is three, three minus two is one, three minus one is two—they knew the arithmetic of the numbers one, two, and three, addition and subtraction. Although that in itself is interesting, what was really intriguing, for those of us that saw this announcement, we said, “How on earth could she find that out?!” Well, you give them a math test is how you find that out. And what she did, she had a very ingenious method of giving them a math test. Very briefly, what she did was she gave them arithmetical sums—like one plus one is two—but not written on paper, in fact not using language at all because these kids don’t yet have language. It was acted out. She had a puppet theater, and the child would sit on the mother’s knee looking at the stage, an empty stage. A hand came on and put a puppet on the stage, so the child sees an empty stage and sees one puppet. The screen comes up to hide the stage, and then the child sees another puppet go on and it goes down behind the screen. So the child sees “one plus one” but there’s a screen there to hide the answer. So the child’s watching—and the child, by the way, is being filmed, the camera’s trained on the child’s face. In particular we’re going to watch the eye dilation and size of reaction in the child. So the screen’s going to come down after the child’s seen “one plus one,” but sometimes—and this is going to be performed many, many times—sometimes an assistant is going to add a puppet or take a puppet away. So the child sees “one plus one,” but sometimes the screen comes down and it sees two. And the child sort of looks at it. However, if the screen comes down and the child

sees three or one, HEY! [Laughter] Eyes dilate! “Should have been two! Should have been two!” They can’t say it, they just know what two is. It knows there should have been two there. It’s sort of hard-wired to recognize one, two, and three. I don’t have a good natural selection reason why it should be important to hard-wire exact one, two, and three—you can make all kinds of stories, but who knows. But it seems that all of us are hard-wired with one, two, and three. By the way, other experiments performed brought it down to two days after birth, which is as young as you can do this sort of thing, with sucking reflexes, skin conductors and things. So young children, all of us, are born with some capacity for numbers, and I’m going to leave you with a story before we take a break, to show how this capacity can manifest itself later on.

One of the stories I tell in the book is of some researchers who went out to Recife, in Brazil, some years ago, about twenty—well after John Lennon had died. [Laughter] They went out into a street market. Now this was one of these noisy street markets you get all over Mexico and South America and the Mediterranean, with bananas and coconuts and papayas and straw hats and cheap plastic sandals and all that kind of stuff. Noise, flies, bustle and everything, and the experimenters were posed as shoppers—the only difference was they had tape recorders in their bags or pockets. They went up to these stalls, they would pick a stall that was being staffed by a young child aged between eight and fourteen. These weren’t street urchins, these were regular kids, in school during the school day, learning math and whatever they learn in the Recife school system. So they were doing regular schooling, but while their parents were away, for lunch or whatever, the kids were looking after the store. The experimenters went up to the stalls and bought stuff. But they bought unusual things, you know, like thirteen coconuts. And they paid with very large denomination bills. So these kids were forced to work out the change, without calculators, to some very difficult subtraction sums. You know, I’ll give you 1,000 cruzeiros for thirteen coconuts—most of us would blanch, I certainly would. But these kids just sort of went, yep, [indicates no problem]. But the experimenters would very often say, “Wait a minute, I don’t think that’s right. Can you tell me how you got that?” And they would force these kids to verbalize their reasoning—it’s 150, we’ll put 100 to one side, subtract 75 from that—they would break the numbers up, do all kinds of complicated things, put them back together, and give the right answer. They were not using the methods they learned in school. They were using methods they had somehow acquired just by watching their parents and other people do it and figuring it out for themselves. They hadn’t really been taught this explicitly. They had learned it by being at the market and working alongside their parents.

The sting in the tail was this: after they’d done this, the experimenters came clean and said they were doing a study, introduced themselves to the parents, arranged to go to these kids’ houses a couple of days or weeks later, to spend an evening talking to the kids—part of which was to give them a little arithmetic test. A classic, standard arithmetic test written down in symbols, you know, you draw the numbers and ask them to subtract them out and things. The questions that they were going to be asked to do were the very same questions that had just been presented in the market stall. The difference was there was no noise, there was no bustle, it was quiet, they had paper, pencil, take as long as you like, nothing’s important, nothing matters, no pressure. Same questions. In the market stall the accuracy rate of these kids was 98%. That was the median accuracy. The median results when they got the same questions using paper and pencil was 37%. [Stunned murmurs] Can these kids do arithmetic? You bet they can do arithmetic, 98% accurate. Can they do pencil and paper arithmetic? Not at all. In fact some of these kids couldn’t multiply by ten. If you said, “What’s 17 times 10?” they would get it wrong. Because they weren’t thinking in terms of the decimal number system, and oh, you just put a zero on the end. They were dealing with real numbers, if you like, the actual numbers of coconuts and money. They weren’t dealing with abstractions.

Okay. I can draw a conclusion from that. You may want to draw your own conclusion, so why don’t we take a break and then after the break we can draw conclusions, we can talk about the book, or about

anything else that's vaguely connected with mathematics and the things that have happened since John Lennon died. *[Laughter and applause]*

Gallin: Before we break I just want to let everyone know that I've read, personally, *The Math Gene* and *The Math Instinct*, and they're both really great books. In fact I got *The Math Gene* for my mother as a gift three or four years ago and then I immediately borrowed and read it. I think I'll get her *The Millennium Problems* this year. *[Laughter]*

Also, mark your calendars for December 20th. We're having two events this month and December 20th is going to be the first ever, or maybe first annual, Holiday Puzzle Party—a friendly competition of math and logic puzzles with *actual prizes*. So see you there.

-----BREAK-----

Gallin: Okay, let's get started again. And for anyone who was afraid to get up and lose your seat during the break, you can come up and look at the bookstore afterwards. Just ask for Christina, who's sitting right there. And now, let's get back to the math instinct. *[Applause]*

Devlin: Okay, questions. I mean, I have all sorts of things I could talk about—you get me going and it's hard to stop me...

Attendee: The desert ant—is there any understanding of what the neurobiology is?

Devlin: I don't think so. I don't know of any.

Attendee: So we don't know how it does it.

Devlin: No. But we don't know *we* do it either.

Attendee: But we know how the silicon chips do it.

Devlin: Yeah, I guess we do.

Attendee: There's been a lot of discussion about human males and females having a different capacity to deal with mathematical concepts—

Devlin: Well it's mainly in Cambridge, Massachusetts, but yeah. *[Laughter]*

Attendee: Do you think there's any validity to the notion that men are better at mathematics than women?

Devlin: No. I mean, there was this stuff that Lawrence Summers did at Harvard, and I think, quite justifiably, got into hot water for saying that. First of all, mathematical thinking is this whole spectrum of thinking. And at one end of the spectrum you have mathematicians who think incredibly spatially and at the other end of the spectrum you get mathematicians who think incredibly linguistically and symbolically. And most of us are somewhere in the middle and we sort of learn to compensate. You know, there've been various studies that show that there are some slight gender differences on the aggregate on various kinds of reasoning, but first of all, they wouldn't manifest themselves in mathematics because it takes care of all of those things. And *even* if there is—and I find it hard to see why natural selection would have made any difference here, not least of all because mathematics came very late in the game. So mathematical thinking,

per se, is not a product of natural selection, it's the mental capacities that we bring together to do mathematics that come out of natural selection. But mathematics itself is something that we've constructed out of those ingredients. Natural selection gave us the ingredients to bake the mathematical cake, but people baked that cake and discovered how much to mix together of the ingredients over the last [inaudible] thousands of years or so. What I'm talking about now is the "math gene" and what [Devlin's book] *The Math Gene* was about was saying, well yes, mathematics is genetically inherited in the sense that the ingredients are a product of evolution, but mathematics itself is a new cake we've learned to bake out of those ingredients.

Back to the gender issue, some of those capacities you might be able to tell an evolutionary story where one gender might have a slight advantage. In terms of mathematics, I think that so many things come together that any of those, if there were any, would sort of balance out. And the reason I got upset with Lawrence Summers wasn't because there may be some scientific fact buried in there somewhere, but everything weighed out suggests that the differences in performance is 99.9% social and cultural. And if we want to attack gender inequality in mathematics it ain't by looking at biology, it's by looking at society. I mean, there's a soft target there. Hit the soft target if you want to do something. Don't worry about whether there's a 2% difference that's genetically inherited.

Attendee: With the example you brought up with the children, the coconuts, the written tests and whatnot, do you think perhaps that there's any relationship between that and people who are considered dyslexic?

Devlin: I've got a dyslectic daughter, so I'm sort of aware of how these things can factor out. In the case of dyslexia there are certainly some parts of mathematics which are hugely hit. I mean anything that's vaguely algebraic and symbolic is clearly going to be hit big in the case of a dyslectic child. However, there's a ton of mathematics that's incredibly spatial, and an awful lot of kids who are dyslectic—the brain has this compensatory thing—they can tend to be incredibly rich in their spatial reasoning. And so you can certainly get dyslectic mathematicians, those who are severely dyslectic but can be brilliant mathematicians. In fact some of the most brilliant mathematicians of all time may well have been dyslectic. Henri Poincaré comes to mind. They are incredibly visual thinkers, they can create in their minds incredible mathematical landscapes that they can explore. And maybe some of the great mathematicians would never have achieved their greatness had they not had some linguistic problems that sort of forced them in one direction. So, I mean, dyslexia and language things will certainly hit some parts of mathematics, but I think there's enough in the mathematical domain that even for the child that's severely dyslectic, there's a lot of mathematics to play with. Now whether they can survive the school system to get to be able to do it is another matter. *[Laughter]* Because the school system is very much—you know it's a square peg, or a round peg, whatever it is, you've got to get through that peg. And we almost certainly lose an awful lot of potential mathematicians because they can't get through that peg shape in order to blossom at the other end.

Attendee: I've been thinking about the six-month-old, and also the beetle, and one other possible explanation is that the six-month-old has three slots that they're filling with images of what they're seeing and they're matching up against what they're seeing. Is that still mathematics even though they're not thinking of numbers in terms of one, two, and three? And with the beetles I was thinking—

Devlin: Not the John Lennon Beatles, is it? *[Laughter]*

Attendee: Sorry, not the beetle, yes, the ant. So I was thinking about how when they're trying to find their way back, and so as a human you're finding your way through the city, using landmarks and you're

reversing direction. Would that count as mathematics? I'm wondering how flexible your definition of mathematics is.

Devlin: Well, I'm trying *not* to define it. I'm saying we humans have to interpret an activity in what's the most natural way to interpret it. But in the case of the first one, there were variants of the experiment—instead of doing it with visual things they did it with sound: beeps and computer generated sounds. Another one did it with dots on a computer screen in different configurations. So we know it's not recognition of spatial patterns, because you can compare dots and sounds and so forth. And kids can also tell the difference between three beats of a drum and three dots on a screen. What's the one thing that's constant: number. In other words you can change the representations, and when you've changed those and the kids still recognize something, the only word that's left to describe what's being recognized is the number one, the number two, the number three. Because everything else varies. It's not representations of the numbers. It's got to be *numbers*.

In the break I was talking to some people about how it's very easy to give evolutionary natural selection stories for why it's important to recognize numbers. One of them is that for small groups of creatures a *huge* survival advantage comes from knowing if you're outnumbered. Are there five of those and four of you? Five of you and four of them? I mean that's such a huge survival advantage that that's going to be very rapidly going into the gene pool. So, recognizing and comparing sizes—doesn't mean to say you have a symbol, some explicit sense of five or six or whatever. But knowing the difference between four of these and five of those, where the five may be sounds you can hear, like lion roars—in fact this was actually done about twenty years ago in the Serengeti National Park. Some researchers went out and they took tape recordings of lion roars. Then they would go out and look for groups of lions and they would count them—so if they saw five lions they would play maybe four lion roars, and the lions just yawned and stayed. If they played five roars the lions would sort of look around a bit. But if they played more roars than there were lions in the group, the lions got up and left. [*Laughter and surprise*] So again, they can't see the five or the six lions, they can just hear the different roars. So what's the constancy between hearing different roars and then recognizing that there are four or five of you in the group? The only constancy we have is what's in common between five of these and five of these and four of these—that's *number*. Number arises when you take collections and you look at matchings between the collections—you know, it's: what do three of these and three of those have in common? What they have in common is “threeness,” or the number three. And once you have creatures that can recognize comparative sizes, especially when the signals are coming in audio or visual or memory or whatever, then the only terminology humans have to describe what's going on is that those creatures have a sense of number. Now it'll be numbers up to the limit of what that creature needs to know, and that's typically three, four, five. There are certain species of bird which certainly have a number sense up into the low teens. You can teach chimpanzees and bonobos to recognize symbols—and I think one or two species of monkeys as well—up into the 30 or 40 range by recognizing the symbols. Although it has to be said that all of the experiments where they teach creatures to recognize numbers require many, many, hundreds of hours of iteration and the creatures never get it 100%. They get pretty good—but contrasted with children, who once they learn about numbers are fascinated by them and, boy, they get the numbers down pretty quickly. And once kids get numbers they realize they could go on forever. The kids only have to understand numbers up to five or six or seven or eight, and then they realize you could keep going and the accuracy of counting comes in. So there's certainly something unique about humans in the number case.

Attendee: I heard that you can't actually visualize more than seven?

Devlin: Yeah, there's a paper by George Miller, a psychologist at Princeton, called something like, “The Magical Number Seven, Plus or Minus One.” To the ordinary person, if you throw down some balls

onto a table or something, anything up to seven you can instantly say how many there are. You recognize three, you recognize five, you can recognize up to seven. Beyond that you have to do what's called subitize, you have to group them and think of them as a group of four and a group of five, or whatever. You have to somehow pull them down to groups less than seven—which you can learn to do very quickly, but it's around seven. The exceptions are these savants, like in the Rain Man movie, where there are these people where you can throw hundreds of things down and they just sort of “see” the number. I find that as baffling as the monarch butterfly, just a weird capacity. But indeed seven is, for most people, where you have to recognize a pattern or group in order to recognize the number. And I find it fun to sort of tell what natural selection advantage would that give—well, families typically probably have five, six, seven kids in them, or that kind of thing. So maybe it was the size of the family where it was important to recognize how many kids you have around you. It's fun to speculate these things—the game really is, with telling these natural selection stories, it's not to tell the right story, because who knows. It's to tell a plausible story. Once you tell a coherent, plausible story, well that's the most you can get.

Attendee: Where does the ability to make logical, airtight arguments come in?

Devlin: Ahh, I should sell you this book, *The Math Gene*. [Laughter] That's what it's all about. That's the book that I wasn't talking about, but I am now, and any author loves to talk about his or her books. *The Math Gene* was a book about how the human brain acquired the capacity to do mathematics. And that took me many years to write because I had most of the story for a long time but there was a big piece of the puzzle that I didn't understand, that I finally solved—actually, by serendipitously coming across some research about how the human brain acquired the capacity for language. As far as we know, about 150,000 years ago the human brain underwent a change—sort of a functional phase shift—that allowed it to do some new stuff. And there's an interesting neurophysiological problem here, as to how a small change in structure gave rise to this phase transition where the functionality increased dramatically. And the story that I tell—and it just piggy backs on some work on the origins of language—is that there are a bunch of capacities that seem unique to humans, as far as we can tell, except in limited forms. One is that we can think outside of time. We're locked in time physically, but we can step outside, think about the future, think about the past, speculate about alternative futures, we can counterfactually speculate about what might have happened in the past. So we can step outside of time and see time from outside, as it were. We can discuss things, we can talk about things, that are not in the environment. By using language I can talk about the other side of the city, about what's going on on the East Coast, what went on two or three years ago, and so forth. So we can not only *think* outside of time, but we can *communicate* outside of time, and that means we can plan.

There's also what you might call *offline thinking*, which means we can run simulations of the world without needing any inputs or any outputs. We don't need any physical stimuli going into the brain—in fact the trick is to close down the stimuli, think about something, and then eventually switch on the [inaudible] and put it on at the end. That's how we do mathematics and lots of other things.

So we can think offline, we can think out of time, we can discuss things outside of the environment, we can think of things outside the environment. All of those capacities, if you think about verbalizing them using the larynx to create sounds, it's equivalent to language, because language is just the articulation of putting things together in the mind. If you can think about various things in a structured way, if you have a way of vocalizing them, then you have language. And mathematics is another one. So there's a lot of work I stumbled across about the connections between offline thinking, thinking out of time, planning, coordinating, use of language—that the guys that were interested in natural language origins had pinned down to about 150,000 years ago, give or take 50,000 years. [Laughter] So somewhere between 100,000 and 200,000 years ago, somewhere in there, they think this happened, based on artifacts and

things. And that led to everything that sort of made the human—that was the unique thing that pushed Homo sapiens along the path.

And so the way I tell *The Math Gene* story is I make the case that the capacity for language *is* the capacity for mathematics, they're one and the same, two sides of the same coin. It's not that one depends on the other, they are actually the same capacity. And I describe in *The Math Gene* how it is that they're the same capacity just manifested in different ways, or ways that appear different to us. And then what happened when that change came along was interesting. First of all, language cannot have come onto the scene in order to communicate. That's what we're using it for. But it's like the computer—what do we use computers for most of all? We use them to communicate, but that's not why they were invented. So once language came onto the scene, boy was it useful for communication—and people used it for that all the time and that's been the driving force. But that cannot, just for simple logical reasons, be what led to it because why do you need language? Well, you need language to communicate complicated ideas. But you've got to *have* them before you can communicate them. So inside the brain there has to be the language capacity, because you've got to put those ideas together before you vocalize them to some other creature. The language origin story has to be that what humans were doing was learning to avoid danger. You look at humans—we're now probably going to destroy the planet because we're the most populous and various other things wrong. We're not good for the planet, it would appear at the moment. But what's led to that domination, or this ability to sort of destroy things, is the ability to coordinate and plan. Because we're not the fastest, we're not the strongest, we don't have sharp teeth, sharp claws, we're not big, we don't have strong armor—our one survival trick is we can anticipate the future and collaborate and cooperate. Thinking offline was our unique trick. The desert ant did its dead reckoning, we do offline thinking. We plan, we coordinate—that surely is the driving force for language. So what language was was the brain learning how to put ideas and concepts together and to create “what if” scenarios. But once you attach symbols to these things that have been put together, and you vocalize them using the larynx, then you've got language. And once you've got language, boy, you can immediately use it for communication.

When that step was happening, all the pieces were in place for mathematics but there was no use for it. Society had to reach a level of complexity where mathematics was needed. And when it came along with the Sumerians in 8,000 BC, the brain was sitting there ready to do it and it did it. So that's the story I tell in *The Math Gene*.

Attendee: What are your thoughts about Steven Pinker? It seems that you would probably agree with a lot of what he says about the origins of language and so on.

Devlin: Yes, yes. I find myself in sympathy with most of what he says.

Attendee: If I may comment. When I drive I notice that my eyes are calculating. I'm doing a lot of calculations. I wouldn't call it math, per se, but my eyes are calculating distance, and something is calculating speed—I'm slowing around the curves, speeding up coming out the curve—and it's like this data is coming in to me and it's in a black box that's all being mixed, weighed, evaluated. And it's the same with my dog, she knows when something is missing, or when there's something extra. And she does it from the same place that I'm talking about.

Devlin: Yes, the visual system of humans and dogs and other creatures embodies huge amounts—in fact I've got a whole chapter of the book about vision. In fact, in Steven Pinker's book *How the Mind Works*, about half of it is about visual systems. And the visual system embeds enormous amounts of complicated mathematics. For example, one thing that you will share with your dog, and with a baseball outfielder, is that if someone throws a ball—and that's why there's a picture of a dog running to catch a

frisbee on the cover of my book—you would think that what you're doing, since you want to reach it before it falls to the ground, is to run in a straight line to where that ball's going, because that gets you there fastest. But you're not. What you will do is you will run in a curved arc. You're not going to be aware of this. You're running in a curved arc because the mathematical problem of deciding where you need to be—you've got your speed and the ball—and calculating where those two things have to be at the right time is *very* tricky. So you don't do it. The way the visual system works is it's very good at compensating for our own movement when we're traveling. So your visual system compensates automatically for all the various relative motions. What you do when you're running in a curved arc—and this has been done by putting little cameras on people, that follow their eyes—you run in a direction that makes it look as though that ball is traveling in a straight line. That means your body automatically performs a compensatory arc, compensating for the arc of the ball. The mathematics is being done by your visual system because it knows how to compensate for motions. And so you run further, but more accurately.

Attendee: Is the dog doing that too?

Devlin: Yes, the dog does the same thing—well we *think* so. Who knows what the dog does. *[Laughter]* You could certainly put little cameras on the dog, to see where it's looking, and what the cameras will pick up is that it looks as though the ball is traveling in a straight line. And so if you apply the Occam's Razor argument—What's the simplest explanation?—is that since visual systems probably work the same for all creatures, because it's a very good system, then that's probably what's going on.

Attendee: I have a question about your view of the future. Is the golden era of mathematics over? And if it's not, what are the big challenges for mathematics in the next twenty years?

Devlin: Well, that really is a matter of defining what we mean by mathematics. If by mathematics you mean classic geometry, then the golden era was 2,000 years ago. If you mean the kind of mathematics that led to the calculus and so forth—Newton, Leibniz—then it's 17th century, and that was the golden era of *that* kind of mathematics. Now, mathematics has always grown and changed in response to the needs of society. It began with numbers; then it was geometry for measurements and architecture; calculus emerged with the development of science and the need to understand motion of planets and so forth; probability theory was developed in the 17th century initially because of the needs of the wealthy patrons of the gaming tables in Europe *[laughter]* and then it eventually found its way out into society with insurance companies and so forth, for the people who draw up the life expectancy tables and figure out how much your life is worth. So all of these different disciplines—I mean, right now there's been huge amounts of pressure put on to developing Bayesian methods in probability calculations for homeland security, to try to assess terrorist risks. So most of the different kinds of mathematics have come up with the needs of society—although it sometimes filters through two or three layers of mathematics, where one new part of mathematics is driven to emerge because of another branch of mathematics—but it always comes back to the changing needs of society.

As we enter the 21st millennium—I mean the third millennium—whatever it's called—

Attendee: After John Lennon. *[Laughter]*

Devlin: That's right. You could sort of speculate as to what the needs would be. The 19th century was the century of chemistry. The 20th century was the century of physics. Now it's one of these clichés that is probably true, but this century is the century of biology and biological systems. Most of the mathematics we now have was developed in response to physics. And it's *really* good at dealing with physics. I think we've been pretty darned lucky that we can apply some of that mathematics—*so well*—

to biological systems, to human systems, to economic systems, to the stock market to a certain degree. I mean, people have won Nobel Prizes in economics for applying mathematics of fluid flow to the valuations of derivatives and so forth. So we've taken this mathematics that's largely in response to the physical universe—of which it's incredibly good for doing physics and engineering—and we've found it's been pretty successful for dealing with the social world and the biological world. But if I look at the way it's applied in the social world and the biological world, it's damned crude compared to the way it fits like a glove in the physical world. Well of course it's crude, because it was developed for the physical world. So I would think that what we're going to see in this next hundred years is mathematics, maybe radically new kinds, developed to study the biological world and the social world.

Now already there's some of that. There's a whole area of mathematics called *chaos theory*, or *dynamical systems theory*—which really only grew up in the 50s and 60s and onwards—which uses a mathematical approach to study dynamical systems, biological systems, the living world. In fact, it's achieved sufficient success—one very good friend of mine, Stephen Wolfram, has written a book called *A New Kind of Science*. He thinks that that approach is really so significant that it amounts to a whole new way of doing science. I don't agree with him that far, but I can see his point, for sure—that the trick of applying mathematics comes from deciding how to look at the world. I mentioned probability theory. Prior to the 17th century, people thought that you could not use mathematics to talk about things that happened randomly, like rolling dice. Because by definition, they said, if something's random it's not mathematical. Mathematics predicts things and random events can't be predicted. Now, that's correct, but what mathematicians in the 17th century and onwards realized was that if you take not one event but a whole number of repetitions of the same action, like rolling dice many times, then there is a mathematical pattern. It's so precise that the casino owners can guarantee weeks in advance what the profit will be the next week. Casinos are not gambling—they know exactly how much they're going to make because of the way the laws of probability work. So the trick to developing probability theory was to look at randomness not as a single event, but as what happens when you apply that single event many, many times. And of course the reason the casinos win is because they have hundreds and hundreds of people playing games around the clock. If they were only playing one or two people then they wouldn't win with the same predictability.

So the trick was looking at it the right way. Dynamical systems was the same thing. You say, well, lots of things in the world look chaotic, but the chaos [inaudible] is simply because you've got something very simple happening many, many times and influencing the next iteration. You've got a feedback loop. So it's not the same as probability where the same thing is repeated. In a dynamical system something simple happens and then depending on the result, something else simple happens, but it takes account of what's happened before so you've got a feedback mechanism. And once people realized that that gives you predictability, then that gave rise to this branch of mathematics called chaos theory. Chaos theory, by the way, does not study genuinely chaotic systems. It studies systems that are actually totally predictable, but which behave in a way that humans find unpredictable. So the predictability is hidden. And the mathematics unravels that hidden predictability just as the mathematics of probability theory unravels the hidden predictability of things like rolling dice. The point I'm trying to make is that the key to making a step forward mathematically is deciding how to look at the world. I don't think we yet know how to look at the world of social activity. We can apply statistics so we can predict what a crowd will do. And we can predict outcomes of voting, elections and so forth, pretty well. But that's sort of crude. I don't think we yet really know how to look at social structures to do mathematics—and maybe we never will. And likewise with biological systems—we have some stuff, but I think we're not there yet. So my prediction, if you like, would be that within the next hundred years someone will figure out how to look at biological or social systems so that we *can* do some mathematics. And it'll be interesting! Because as far as we can see, people are unpredictable—and yet they're not. There's an

awful lot of evidence from sociology that shows that people behave in remarkably predictable ways, we just haven't got the handle on it yet to do mathematics.

Attendee: Do you think a machine will pass the Turing test?

Devlin: I don't think a machine will pass the Turing test. I think that's just not going to happen. The Turing test, by the way, is: can you develop a machine that, if you were interacting with it using your natural language—talking with it or typing into a keyboard—can you develop a machine where when you try to interact with it, a human being could not tell whether it was a machine or a person. Now, it's sort of a thought experiment to a large extent, but even at that level the reason I think the answer is no is I don't think people are rule-based. I think we can apply rules to how thought goes, and the rules are sort of descriptive but they're not prescriptive. Whereas computers—and I'm talking about digital computers, if we start talking about assemblages of neurons and biological tissue then all bets are off—but if we're talking about digital computers that are run by a program, I think, fundamentally, digital computers operate in a very different way from the human brain. One is predictive and rule-based and the other is, for want of a better word, biological, or organic. So I think the Turing test isn't going to be passed by a digital computer, but whether it'll be passed by something we build is another matter.

Attendee: Your friend Stephen, what was his point of view? What were you starting to say about that?

Devlin: Stephen Wolfram? He thinks that we're on the verge, or that we've actually passed the verge, of a new kind of science. His book is about this thick [*indicates around 4 inches*]! He takes this observation that we look at the world, we look at flowers and trees and hurricanes, and they look very complicated. A classic example is a tree. A tree, if you look at it, looks incredibly complicated—there are all these branches and twigs and things. But here's what you can see with a tree—when you look at a tree from fifty feet away you see a trunk and some things coming out of it with some little green dots on it. Then you get a bit closer, and instead of looking at the trunk you pick on one branch and you forget everything else. And what's that branch—well, it's a long thing with some thinner things coming out and some little green things on the ends. Then you look at one of *those* twigs and you see a straight thing—and you say, wait a minute, the complexity is actually just a juxtaposition of something very simple: something growing up and three things growing out of it. And three things grow out of each one of those, and each one of those, and so on. So you can take a simple rule and if you apply it over and over again, emergent complexity can result. The complexity we see in the living world doesn't mean that there are complex rules. All that you need to [*inaudible*] Occam's Razor is that there is a simple rule applied many times. And what Wolfram says is, look, science uses all this machinery of mathematics, complicated differential equations, looking for what are intrinsically complicated explanations of natural phenomena. So Wolfram's point is—and he could certainly make a much better job of this than I can—that the way to look at the universe is in terms not of relationships, which we describe by equations, but in terms of processes, which we describe by algorithms. So he says we should forget relationships and equations, we should think of algorithms and programs—and try to do all the science that we can thinking in terms of little algorithms that would run things and produce the world.

Attendee: What about interactions and feedback loops—

Devlin: Feedback loops he's got. Interactions is another issue—

Attendee: What about self-modifying learning systems?

Devlin: He's got that. Yeah, anything that's a process in time, he's got. I buy all the stuff, I just don't think there's a “new kind” of science there. There may be a new kind of science that comes out of it—

Attendee: [Inaudible]

Devlin: It's really hard to see the future. What we're saying is that you should expect the unexpected, I guess. We're trying to look forward and see what could we imagine that mathematics would be that would really capture biological systems well, or the brain well, consciousness, or the social world, for example. And yet, because we are trapped in today's scientific frameworks, we have to try to extrapolate from there. I think that if history is any lesson, it'll involve a completely new way of looking at things. Once that's on board, people will look back and say, "How come those guys in the year 2006 couldn't see it? It was staring them in the face!" It was the same thing with relativity. People worried about simultaneity and time, and then Poincaré almost got it but not quite, and then Einstein came along and said that time is light and it's all to do with signals traveling—and really just looked at it in a different way. We can look back and read Poincaré who *almost* got it in his writings, and we can say, "You were so close! You were this genius, why didn't you see it?" Well, because it's easy to see with hindsight.

Wolfram may be close, I'll grant him that, but I don't think he's there. Someone will later come along and then we'll look back and say either Wolfram was the first or he was *so close!* I think that's going to happen, but if you ask what will it look like—I have no idea! If I did I'd be doing it.

Attendee: A lot of the examples I've heard of mathematics in natural systems, they tend to fall into certain categories in mathematics. I was thinking, there are certain branches of mathematics, like group theory, that I can't think of instances in the natural world where a system displays that kind of thing—

Devlin: Except you're made up of systems that do display [inaudible]. [*Laughter*]

Attendee: Well, I was just curious, you were talking about maybe trying to find some new ways of describing these more complex systems. Is that where you think we should start?

Devlin: First, let me mention that a large proportion of the Nobel Prizes in physics—that went on small-scale physics, subatomic physics—what it took to get the Nobel Prize was to write down a correct group. If you look at most of the Nobel Prizes, what they did was they formulated a group that described certain subatomic structures. So when I said that you're built up of systems where group theory's important—group theory is an abstract branch of mathematics, which you can see manifested in symmetries like the symmetries of flowers and so forth, simple examples—but that's so trivial it's not group theory. Group theory turned out to be very important in physics, but it was the physics that we couldn't see with our minds—we can partially see with powerful telescopes, but we can really see through the eyes of mathematics. It's the equations that get written down that allow us to see the fine grain of the universe, and that mathematics is built upon group theory. I often say that the most powerful manifestation of mathematics is that it allows us to make the invisible visible. You look outside and you watch an airplane flying overhead. With your eyes you cannot see what keeps the airplane in the sky. To see what keeps it in the sky you need mathematics, you need to look at the equations that describe what keeps it up. There are forces keeping it up, but you need mathematics to describe them. You can't see the radio waves, or the microwaves that go between one mobile phone and another mobile phone via the groundstation. It takes mathematics to see what's going on. So mathematics is like a visualization technology that allows us to see certain things.

Attendee: It seems that math functions as our way of seeing the universe in the same way that we use science—that we can have evidence and we can have data, but without a theory it's meaningless. So mathematics gives us the forum that allows us to make sense of things.

Devlin: Oh yes, one of Galileo's famous quotations—or misquotations, because he didn't quite say it this way—was, “In order to understand the universe you have to understand the language in which it's written, and that language is mathematics.” Now there's a sort of a triviality to that. You've got to remember that what we think of as science is the stuff that works, and that was successful. The history of science is the history of the successful bits, which is the 5% that works and not the 95% that was wrong. Most of the stuff that Newton thought about was related to gold, and alchemy, and stuff that we would now regard as fringe stuff. He's remembered, of course, for the stuff that lasted, like calculus and optics. So what we think of as science is the stuff that turned out to work. If you look at any stage of science and you look at what they're doing at the time, it's often very difficult to tell whether it's going to be lasting or not. The classic case to this is string theory. It hasn't been tested, it may never be tested, there's no doubt that the guys working on string theory are unbelievable smart, they're doing very clever stuff—but fifty years from now will that be like the flogiston theory, or various theories that we now regard as nonsense? Will it be regarded as just a big clever mistake? It's very hard to see how history will turn these things.

Nowadays, we look at mathematics and how it helps us to understand the universe, but mathematics helps us to understand those aspects of the universe for which mathematics turned out to be useful to help us understand them. *[Laughter]* Well, yeah, duh! It works there because it was developed to work there! Can we apply some way of thinking that will apply to social structures, or to the brain and consciousness, or to biological systems—will we develop some way of thinking about them that in a hundred years from now we'll call mathematics? Or will we call it something else, or will there be something else? I don't know. All we know is that hitherto, thinking in certain ways about things has led to certain insights that have led to stuff that we've classified as mathematics, and there's no reason to assume that's not going to continue, but we really don't know.

Attendee: I've heard about lobsters on the east coast, that you can just plop in the ocean and they'll find their way home. *[Inaudible]* the earth's magnetic field. *[Inaudible]*?

Devlin: We know that bird migration—migration of lots of creatures—they use a combination of things. Certainly many of them, if not most of them, use the magnetic field. They also use the position of the sun in the sky. They use the polarization of the sun, sunlight on a cloudy day. Many species of birds and whales and fish can read the stars. They can go by moonlight, polarization of moonlight. And most of these creatures have fail-safe devices—and again, natural selection has built these in—so that if it's cloudy, or if it's night, they have compensatory systems that they can use, and they use combinations. There have been all sorts of experiments with fish in tanks—in the case of lobsters they took them out of the ocean, put them in a tank of water, took them onto land, put them in the back of a pick-up truck, drove them past magnetic fields, disoriented them, took them back to the ocean, put blinkers on them and plopped them back in the water—and these little creatures went back to their home. Now, the only explanation the guys have, and it's a partial explanation, is that they can sort of see the earth's magnetic field. Not just that they can *sense* it, but they can *see* it with such precision that they can find their way home by negotiating the magnetic field. With lobsters, and with birds, if you put a little magnet on their head, they've had it. If every direction is north, you fool them. And I think bees and dung beetles—which are the two examples that have been studied that I'm aware of—if you put a polarizing filter over them they stop flying. They need the polarization of the light, which is always there and it's a good thing to rely upon. Although that's pretty complicated, because if you're using light from the sun or the moon, be it direct or polarized, both the sun and the moon's position in the sky change during the course of the day and during the course of the year. The mathematics involved in navigating by sun or moon is unbelievably complicated in human terms. So creatures that migrate, their built-in instinctive systems are doing what in human terms are pretty sophisticated calculations. Do you call it *calculation*?

Probably not. They're just doing their thing. But that amounts to calculation in the same way that a calculator's doing calculation.

Are we out of time, or—you're the boss!

Gallin: Well, how do you feel?

Devlin: Oh, I'm fine.

Gallin: Want to take—how about two more questions.

Attendee: Can you define what you mean by an algorithm? What I understand as an algorithm is a series of calculations that evaluate to true or false or a quantity, and conditional branching on that.

Devlin: That's a pretty precise notion that's sort of designed for computers. I always say an algorithm is just a sequence of steps to be followed. Like an algorithm for how to repair a bicycle tire is take off this, take off that. Or a recipe, or a knitting pattern—knitting is a good one because you can represent it symbolically. But an algorithm is just a sequence of steps to be followed.

When I said earlier that the brain acquired the ability to do mathematical thinking and language, another way I could have said it was that the brain learned how to create and run algorithms. I didn't mention that then, but that's another manifestation.

Attendee: The reason I brought that up is that I personally think that computers will pass the Turing test. I think that the brain has a lot more than two states and a lot more rules, I mean it's just orders of magnitude greater complexity than even networks—

Devlin: Right, so the question is: is it just orders of magnitude greater or is it something different.

Attendee: Well, there might be more than two states—not just on or off, but say, on, off, and green or something like that.

Devlin: Right, if it's a finite number of states then the game is on the Turing test passing. If the brain is doing something else, like a continuum of [inaudible] states, then you're in a different realm. Insofar as you're talking computation you're talking continuous computation, which is a different animal. My instinct tells me that the brain is not digital. Whether it's binary or ternary or whatever, I just don't think it is finite in that sense. But of course I don't have any evidence of that.

Attendee: Do you think it's more analog than quanti?

Devlin: Totally analog, yeah. Whatever that means. And I see lots of evidence, but then again I'll look at the phenomenon like the animals I describe and I'll say, oh yeah, what they're really doing is they're doing something analog and then they're just sort of chopping off the answer and that's why they sometimes get the answer wrong, because they chop it off wrongly. But they're probably accurate on an animal scale because it's important to them. But then again, I would see it that way because that's the way I think about it [*laughing*]. So I'm going to see the world as analog because I've just started to think of it that way. I think the safe thing about making these claims about the Turing test is I don't think it's going to happen in my lifetime.

Attendee: What would it mean to you if it *were* passed?

Devlin: I'd be excited!

Attendee: But what would it mean? Would it change your perspective on what the mind is, or the brain, rather?

Devlin: It depends—the classic example was in the early days of computers. People thought, what's the thing that to humans is the most challenging intellectual problem? Chess. It's the epitome of intelligence. So they said, if a computer could ever beat a human being at chess, boy, will computers be smart. Computers came along, they eventually did it, and people said, wait a minute! The computer's not really playing chess it's just evaluating millions of positions. Well, if we'd known that we would have Moore's Law iterating many years, with our fast processors, masses of memory—all that happened there was we learned that chess doesn't require an intellect to play, it just requires computational speed and a huge amount of memory. And so we said, well, it's a different game. It's like saying, I bet I could run a hundred meters faster than you—and then I turn up with a Harley Davidson., And you say, wait a minute! *[Laughter]* That's not fair!

Attendee: So if the Turing test were passed would you say we need a Turing test II?

Devlin: Well, at the moment it seems to hinge upon natural language. A computer that can manipulate natural language so well that it's hard to tell whether it's a computer or not. I think certainly what can happen—and this maybe plays into the chess thing—is that it's remarkably easy to fool people, by just small amounts of things. In Japan they've got these robots. You walk into a room—I saw this in a lab in Japan—and in the corner there's this robot. And you can see all the machinery. You can see all the insides and the cables and everything. It's got two cameras that are looking out, and above the cameras there are a couple of bars that are like eyebrows, that raise and lower. That's all it's got. And you can follow its eyes and see the lens focusing, and you can see the eyebrow going up. You think you're being looked at by a sentient being. All it takes is that thing to raise and lower its eyebrows—and you can see all the machinery—and its eyes following you, and boy, do you think you're communicating with a person. You just get the same feeling you're being followed as if it's a person. In other words all of your evolutionary background tells you that if something follows you with its seeing mechanism and moves its eyebrows, it's sentient. So passing the Turing test, in some sense, we can already do it in that we can create the sensation that you're dealing with a sentient being. You can start playing with the Turing test and maybe you'll instances where on some level you'll pass. But this is like the chess game again, because what you're doing is you're figuring out what it takes to fool a human being. The original Turing test was: can the machine simulate intelligence. And you're pinned down to natural language. That version is the version I don't think will pass. But if you're allowed to play other games, where you can play with people's emotions, it's a different game. But once you start allowing the game to be played differently, interesting things happen.

[Sorry readers, tape cuts off here, five minutes before the end of the talk.]

END

Be sure to check out Keith Devlin's many books. Here are just a few titles:

The Math Instinct: Why You're a Mathematical Genius (Along with Lobsters, Birds, Cats, and Dogs)
Thunder's Mouth Press (2005)

The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time
Basic Books (2002).

The Math Gene: How Mathematical Thinking Evolved and Why Numbers Are Like Gossip
Basic Books (2000).

Mathematics: The New Golden Age
Columbia University Press (1999).

The Language of Mathematics: Making the Invisible Visible
W. H. Freeman, (1998).

Life by the Numbers (accompanies the PBS TV series by the same name)
John Wiley (1998).

Goodbye Descartes: The End of Logic and the Search for a New Cosmology of the Mind
John Wiley (1997).