

Ask a Scientist Pi Day Puzzle Party

3.14 2016

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1. STAIRS

Consider a staircase of nine stairs, which you can climb by taking any combination of one or two steps at a time.

QUESTION: What is the total number of ways in which you can climb the staircase as described? (e.g., for three stairs the number of ways would be 3, namely: 1+1+1 or 1+2 or 2+1.)

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2. JOE'S FEED STORE

Joe runs a feed store. One day Larry came into the store to buy 13 pounds of feed.

Larry looked around the store and noticed that Joe didn't seem to have an electronic scale, just an old balance scale. Larry asked whether he could really get exactly 13 pounds. "No problem," said Joe, patting the rock sitting next to his prized and polished balance scale. "We pride ourselves on the fact that you can get any quantity you want, so long as it's in one pound increments."

"Don't tell me that's a magic rock that can suddenly become 13 pounds," said Larry. "Nope, just brains and brawn around here," said Joe. "This here rock weighs exactly 32 pounds. Had it checked out by the guys down at the county office. And that's all I need." Joe proceeded to measure out and package exactly 13 pounds for Larry.

QUESTION: How did he do it?

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3. CIRCLE AND SQUARE

I have a square with sides of length 2 units. I also have a circle whose center is the center of the square. They intersect in such a way that the region inside the circle but outside the square has the exact same area as the region inside the square but outside the circle. I.e., the 4 "moon slices" equal the 4 "triangle-like pieces".

QUESTION: What is the radius of my circle?

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4. TATTOOED TRUTHERS

You are placed in a room with a group of five honest football players wearing jerseys numbered 1 through 5. Each might (or might not) have tattoos on his arm. *They* all know who has tattoos and who doesn't. But they're all wearing long sleeves under their jerseys, so that *you* don't know who's tattooed and who's not.

In response to questions, they will each "answer" truthfully, but only by pointing to someone in the room. They will not communicate in any other way. If one does not know to whom to point, he will do nothing. If you ask one a question for which the answer is zero, he will point to you.

You may ask up to two distinct players one question each. In so doing, you must determine which players have tattoos and which do not.

QUESTION: How will you do this? (Be elegant! In other words, there is at least one solution that can be written in no more than a few sentences.)

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5. CUBE SCULPTURE

We construct a sculpture consisting of infinitely many cubes, as follows. Start with a cube with side length 1. Then, at the center of each face, attach a cube with side length $\frac{1}{3}$ (so that the center of a face of each attached cube is the center of a face of the original cube). Continue this procedure indefinitely: at the center of each exposed face of a cube in the structure, attach (in the same fashion) a smaller cube with side length one-third that of the exposed face.

QUESTION: What is the volume of the entire sculpture?

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6. PIZZA

Consider this (actual) math news headline that was written almost exactly two months ago:

“A Scientific New Way to Slice Pizza: Developed by mathematicians, the method serves 12 identically shaped and sized pieces, six with crust, six without.”

Note that the entire pizza must be used; no part of the pizza can be thrown away.

QUESTION: Draw the sliced pizza that fits this description.

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7. MATCHSTICK SHAPES

Suppose you have a box of matchsticks. Using those sticks you want to make the following shapes: an equilateral triangle, a square, a regular pentagon, and a regular hexagon. (Note that in any “regular” polygon, the sides are all the same length.)

The number of matches in the box is such that every possible pair of the four shapes can be constructed using precisely all of the matches. For example, with 7 matchsticks we can make an equilateral triangle and a square. (But, in that case, no other pair.) With 10 matchsticks we could make an equilateral triangle and a square, or we could make a square and a hexagon.

Note that the shapes should be distinct. No two shapes should share a matchstick.

QUESTION: What is the smallest number of matches that could be in the box? (Breaking them is not allowed.)

SOLUTIONS



1. STAIRS

Answer = 55

The answers are fibonacci numbers.

for 1 stair there is 1 way (1)

for 2 there are 2 ways (1+1, 2)

for 3 there are 3 ways (1+1+1, 1+2, 2+1)

and then it seems to get hard.

Note: When faced with k stairs you can start the climb with one step or two steps. If you start with one, it is the same as facing $k-1$ stairs and if you start with two, is the same as facing $k-2$ stairs. Hence the fibonacci number idea (i.e. by induction).

2. JOE'S FEED STORE

The number 32 is particularly nice since it is a power of 2. So Joe proceeds as follows:

First, he measures out 32 pounds of feed using the stone. Setting the stone aside, he then uses the scale to split the 32 pounds in half – 16 and 16. He sets 16 aside and splits the other 16 into 8 and 8 using the scale. Proceeding in this way, he can measure 1, 2, 4, 8, 16, or 32 pounds. To get 13, he takes $8 + 4 + 2 + 1 = 13$.

3. CIRCLE AND SQUARE

Answer = $2/\sqrt{\pi}$. (=1.1283791671)

Note that the area of the circle is the same as the area of the square, which is four.

Since the area of the circle is πr^2 , and since that is also four, the radius is $= 2/\sqrt{\pi}$. (=1.1283791671)

4. TATTOOED TRUTHERS

Gentlemen, please stand in decreasing jersey order, and note the binary number thus created by assigning each of you a one for being tattooed, or a zero for having none.

Then ask these questions of any of the players:

Question 1: What is whole-number quotient when you divide this number by six?

Question 2: What is the remainder when you divide this number by six?

... or, for the even harder-core geeks among you:

Question 1: In base 6, what's the first digit of this number?

Question 2: In base 6, what's the second digit of this number?

Answer #1 times six, plus Answer #2, gives you the number, from which you can extract the required binary information (i.e. which players have tattoos). *Example: if #5 and #3 alone have tattoos, then the binary number generated in the first step will be $16+0+4+0+0=20$, which will yield "3" and "2" as the answers to the two questions. $3*6+2$ yields 20, which is 10100 in binary, which is what we needed to know.*

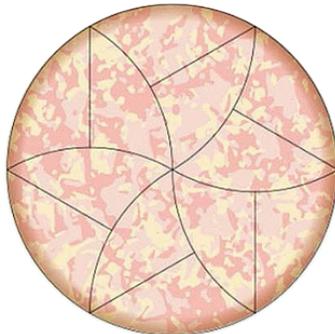
5. CUBE SCULPTURE

Answer = 14/11

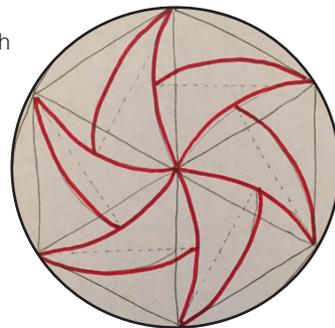
It's a geometric series: The first term is $6/27$; the common ratio is $5/27$; thus the infinite sum is $3/11$. Add in the original cube (volume = 1) for final answer 14/11.

6. PIZZA

Official pizza, which allows congruent ("equal" or "equal with reflection") slices:



Alt pizza, with no reflection:



7. MATCHSTICK SHAPES

Answer = 36

The triangle/hexagon case makes the answer a multiple of 3, and the square/hexagon case makes it a multiple of 2. Hence, it's a multiple of 6.

The pentagon/hexagon case makes it the sum of a multiple of 6 and a (non-zero) multiple of 5.

36 is the smallest number that meets both of those requirements ($30+6$).