11. Two Ferryboats

Two ferryboats start at the same instant from opposite sides of a river, traveling across the water on routes at right angles to the shores. Each travels at a constant speed, but one is faster than the other. They pass at a point 720 yards from the nearest shore. Both boats remain in their slips for 10 minutes before starting back. On the return trips they meet 400 yards from the other shore.

How wide is the river?

12. Guess the Diagonal

A rectangle is inscribed in the quadrant of a circle as shown. Given the unit distances indicated, can you accurately determine the length of the diagonal AC?

Time limit: one minute!

13. Cross the Network

One of the oldest of topological puzzles, familiar to many a schoolboy, consists of drawing a continuous line across the closed network shown so that the line crosses each of the 16 segments of the network only once. The curved line shown here does not solve the puzzle because it leaves one segment un-
21. The Colliding Missiles

Two missiles speed directly toward each other, one at 9,000 miles per hour and the other at 21,000 miles per hour. They start 1,317 miles apart. Without using pencil and paper, calculate how far apart they are one minute before they collide.

22. The Sliding Pennies

Six pennies are arranged on a flat surface as shown in the top picture. The problem is to move them into the formation depicted at bottom in the smallest number of moves. Each move consists in sliding a penny, without disturbing any of the other pennies, to a new position in which it touches two others. The coins must remain flat on the surface at all times.
**Bottoms Up!**

Imagine that you've got a barrel sitting on a lazy-susan and it has four equally-spaced holes around its base. Inside each hole is a glass, and each glass is either right-side-up or upside-down. A move consists of reaching into any two holes, feeling the configuration of the two glasses inside, and leaving them as they were, or reversing one or both of them. You can't leave them lying on their sides or anything like that: this is a pure mathematical puzzle.

After each such move, however, the barrel is spun around so that you lose complete track of which holes were in which positions. Basically, you only have two options for holes: a pair of opposite holes or a pair of adjacent holes, and there's no way to tell which opposite or which adjacent holes you've chosen.

The goal is to have all four glasses the same: either all facing up or all facing down. There is an oracle that will instantly tell you if you have reached a "solved" configuration and the game stops at that point.

Can you provide an algorithm that will guarantee that the game ends in a fixed, finite number of moves?

Obviously, you could keep spinning, reaching in at random, and making sure all the glasses were up, but since you only get to investigate two holes at a time, the downward-facing glass could be missed for any number of spins, so that's not a good solution.

By reaching first into opposite and next into adjacent holes, you can guarantee that three of the glasses will all be up after just two moves, but finding the fourth glass could take arbitrarily long.

I really like this one! Let's plan on using it! It's tough, though. I spent about 20 minutes on it and got maybe halfway through, then bounced it off a friend I consider a puzzling heavy-hitter, and we polished it off in another hour or so of idle thought plus maybe ten minutes of animated discussion. I think teams would especially like this one, and it's challenging enough for dedicated solvers.
Collating the Coins

Even the simplest of household tasks can present complicated problems in operational research. Consider the preparation of three slices of hot buttered toast. The toaster is the old-fashioned type, with hinged doors on its two sides. It holds two pieces of bread at once but toasts each of them on one side only. To toast both sides it is necessary to open the doors and reverse the slices.

It takes three seconds to put a slice of bread into the toaster, three seconds to take it out and three seconds to reverse a slice without removing it. Both hands are required for each of these operations, which means that it is not possible to put in, take out or turn two slices simultaneously. Nor is it possible to butter a slice while another slice is being put into the toaster, turned or taken out. The toasting time for one side of a piece of bread is thirty seconds. It takes twelve seconds to butter a slice.

Each slice is buttered on one side only. No side may be buttered until it has been toasted. A slice toasted and buttered on one side may be returned to the toaster for toasting on its other side. The toaster is warmed up at the start. In how short a time can three slices of bread be toasted on both sides and buttered?

50. A Fixed-Point Theorem

One morning, exactly at sunrise, a Buddhist monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiraled around the mountain to a glittering temple at the summit.

The monk ascended the path at varying rates of speed, stopping many times along the way to rest and to eat the dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed.
Prove that there is a spot along the path that the monk will occupy on both trips at precisely the same time of day.

51. How Did Kant Set His Clock?

It is said that Immanuel Kant was a bachelor of such regular habits that the good people of Königsberg would adjust their clocks when they saw him stroll past certain landmarks.

One evening Kant was dismayed to discover that his clock had run down. Evidently his manservant, who had taken the day off, had forgotten to wind it. The great philosopher did not reset the hands because his watch was being repaired and he had no way of knowing the correct time. He walked to the home of his friend Schmidt, a merchant who lived a mile or so away, glancing at the clock in Schmidt’s hallway as he entered the house.

After visiting Schmidt for several hours Kant left and walked home along the route by which he came. As always, he walked with a slow, steady gait that had not varied in twenty years. He had no notion of how long this return trip took. (Schmidt had recently moved into the area and Kant had not yet timed himself on this walk.) Nevertheless, when Kant entered his house, he immediately set his clock correctly.

How did Kant know the correct time?

52. Playing Twenty Questions when Probability Values Are Known

In the well-known game Twenty Questions one person thinks of an object, such as the Liberty Bell in Philadelphia or Lawrence Welk’s left little toe, and another person tries to guess the object by asking no more than twenty questions, each answerable by yes or no. The best questions are usually those that divide the set of possible objects into two subsets as nearly equal in number as possible. Thus if a person has chosen as his “object” a number from 1 through 9, it can be guessed by this procedure in no more than four questions—possibly less. In twenty questions one can guess any number (or 1,048,576).

Suppose that each of the possible different values to represent the chosen. For example, assume that a one ace of spades, two deuces of spade to nine nines, making 45 spade en shuffled; someone draws a card. You yes-no questions. How can you in questions that you will probably have

53. Don’t Mate

Karl Fabel, a German chess problem outrageous problem.

You are asked to find a move for which an immediate checkmate of the black king...
19. The Flight around the World

A group of airplanes is based on a small island. The tank of each plane holds just enough fuel to take it halfway around the world. Any desired amount of fuel can be transferred from the tank of one plane to the tank of another while the planes are in flight. The only source of fuel is on the island, and for the purposes of the problem it is assumed that there is no time lost in refueling either in the air or on the ground.

What is the smallest number of planes that will ensure the flight of one plane around the world on a great circle, assuming that the planes have the same constant ground speed and rate of fuel consumption and that all planes return safely to their island base?

20. The Repetitious Number

An unusual parlor trick is performed as follows. Ask spectator A to jot down any three-digit number, and then to repeat the digits in the same order to make a six-digit number (e.g., 394,394). With your back turned so that you cannot see the number, ask A to pass the sheet of paper to spectator B, who is requested to divide the number by 7.

"Don't worry about the remainder," you tell him, "because there won't be any." B is surprised to discover that you are right (e.g., 394,394 divided by 7 is 56,342). Without telling you the result, he passes it on to spectator C, who is told to divide it by 11. Once again you state that there will be no remainder, and this also proves correct (56,342 divided by 11 is 5,122).

With your back still turned, and no knowledge whatever of the figures obtained by these computations, you direct a fourth spectator, D, to divide the last result by 13. Again the division comes out even (5,122 divided by 13 is 394). This final result is written on a slip of paper which is folded and handed to you. Without opening it you pass it on to spectator A.

"Open this," you tell him, "and you will find your original three-digit number."

Prove that the trick cannot fail to work regardless of the digits chosen by the first spectator.

21. The Collide

Two missiles speed directly toward each other at 3,171 miles per hour and the other at 5,342 miles apart. With what speed do they collide?

22. The Slide

Six pennies are arranged on a flat surface. The problem is to move the coins to new positions such that they are depicted at bottom in the same way in the picture. The problem is to move the coins to new positions such that they are depicted at bottom in the same way as at top. The coins must remain flat on the surface.
Two Equal Halves
This shape can be divided in half by a single line so that the two halves are exactly the same shape and size as each other. The line may be straight, angled or curved.