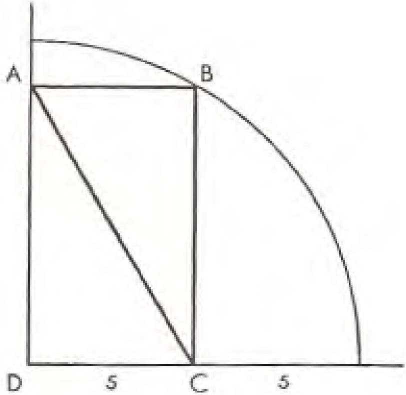
12. Guess the Diagonal

A RECTANGLE is inscribed in the quadrant of a circle as shown. Given the unit distances indicated, can you accurately determine the length of the diagonal AC?

Time limit: one minute!



21. The Colliding Missiles

Two missiles speed directly toward each other, one at 9,000 miles per hour and the other at 21,000 miles per hour. They start 1,317 miles apart. Without using pencil and paper, calculate how far apart they are one minute before they collide.

Bottoms Up!
Imagine that you've got a barrel sitting on a lazy-susan and it has four equally-spaced holes around its base. Inside each hole is a glass, and each glass is either right-side-up or upside-down. A move consists of reaching into any two holes, feeling the configuration of the two glasses inside, and leaving them as they were, or reversing one or both of them. You can't leave them lying on their sides or anything like that: this is a pure mathematical puzzle.

After each such move, however, the barrel is spun around so that you lose complete track of which holes were in which positions. Basically, you only have two options for holes: a pair of opposite holes or a pair of adjacent holes, and there's no way to tell which opposite or which adjacent holes you've chosen.

The goal is to have all four glasses the same: either all facing up or all facing down. There is an oracle that will instantly tell you if you have reached a "solved" configuration and the game stops at that point.

Can you provide an algorithm that will guarantee that the game ends in a fixed, finite number of moves?

Obviously, you could keep spinning, reaching in at random, and making sure all the glasses were up, but since you only get to investigate two holes at a time, the downward-facing glass could be missed for any number of spins, so that's not a good solution.

By reaching first into opposite and next into adjacent holes, you can guarantee that three of the glasses will all be up after just two moves, but finding the fourth glass could take arbitrarily long.

50. A Fixed-Point Theorem

ONE MORNING, exactly at sunrise, a Buddhist monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiraled around the mountain to a glittering temple at the summit.

The monk ascended the path at varying rates of speed, stopping many times along the way to rest and to eat the dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed.

Prove that there is a spot along the path that the monk will occupy on both trips at precisely the same time of day.

20. The Repetitious Number

AN UNUSUAL parlor trick is performed as follows. Ask spectator A to jot down any three-digit number, and then to repeat the digits in the same order to make a six-digit number (*e.g.*, 394,394). With your back turned so that you cannot see the number, ask A to pass the sheet of paper to spectator B, who is requested to divide the number by 7.

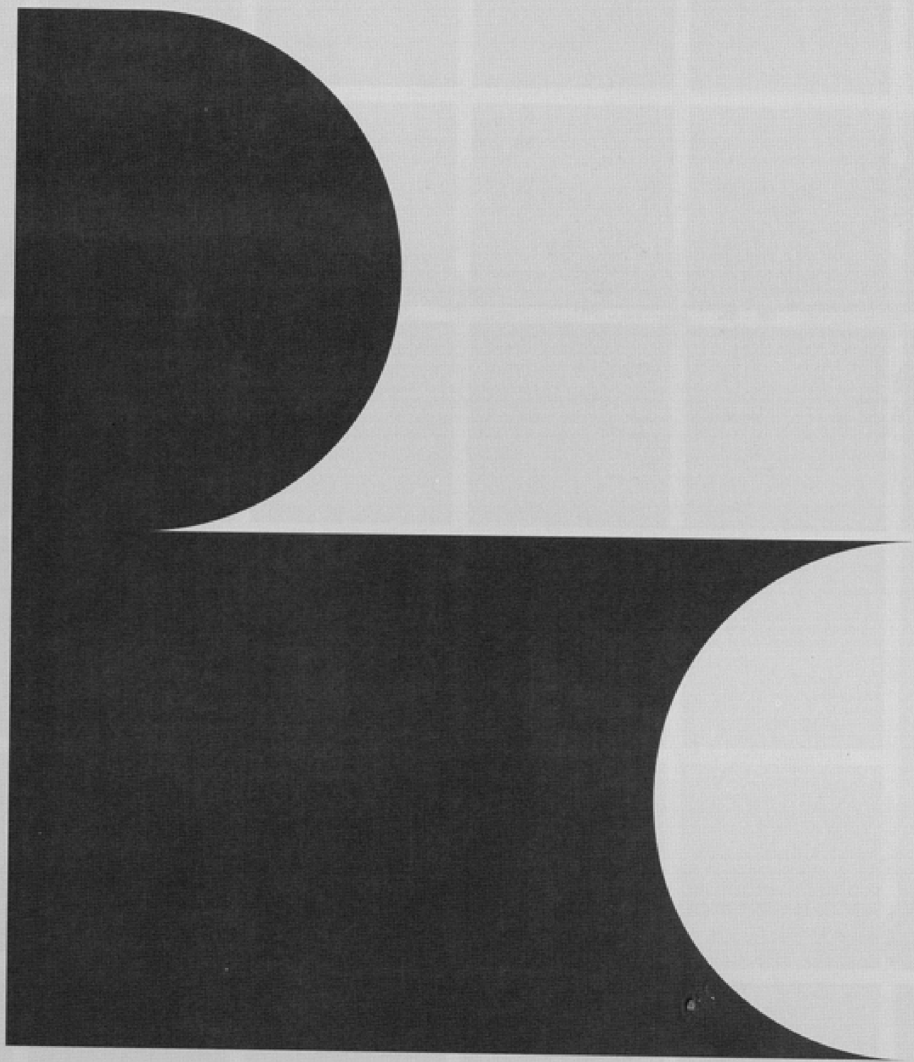
"Don't worry about the remainder," you tell him, "because there won't be any." B is surprised to discover that you are right (*e.g.*, 394,394 divided by 7 is 56,342). Without telling you the result, he passes it on to spectator C, who is told to divide it by 11. Once again you state that there will be no remainder, and this also proves correct (56,342 divided by 11 is 5,122).

With your back still turned, and no knowledge whatever of the figures obtained by these computations, you direct a fourth spectator, D, to divide the last result by 13. Again the division comes out even (5,122 divided by 13 is 394). This final result is written on a slip of paper which is folded and handed to you. Without opening it you pass it on to spectator A.

"Open this," you tell him, "and you will find your original three-digit number."

Prove that the trick cannot fail to work regardless of the digits chosen by the first spectator.

Two Equal Halves



Two Equal Halves

This shape can be divided in half by a single line so that the two halves are exactly the same shape and size as each other. The line may be straight, angled or curved.