#1 - GOLDEN GATE SIBLINGS

Two brothers, riding their bikes together across the Golden Gate Bridge, were 5/8 of the way across when, looking in their rear-view mirrors, they spotted their sister approaching in a car at 30 miles per hour.

They immediately split apart, each biking (at the same speed) toward opposite ends of the bridge. Each arrived at his end of the bridge at the same time as his sister, who continued across the bridge and beyond at a constant speed of 30 mph.

**QUESTION:**
How fast were the boys biking, in mph?

#2 - WATER

You've got an empty 4-ounce glass, an empty 9-ounce glass, an unlimited supply of water, and a catch basin (into which all waste water goes).

Your goal is to end with exactly 6 ounces of water in one glass, and nothing in the other.

**QUESTION:**
What's the smallest amount of water you can have in the catch basin once you've done this?

#3 - THE VERY VERY BIG DIVISION

For every whole number n that is 2 or greater, let’s define the pow of n as follows: The pow of n is the largest power of the largest prime that divides n.

For example, the pow of 144 equals 9, because 3 is the largest prime that divides 144, and if you square it (getting 9), it still divides 144, but if you cube it (getting 27), it doesn’t.

A second example: the pow of 45 is 5, because the largest prime that divides 45 is 5, but if you square it (getting 25), it no longer divides 45.

A third example, just in case: the pow of 1000 is 125.

Now, take the pow of 2 and multiply it by the pow of 3, and then multiply that by the pow of 4, and so on, until you’ve multiplied all the pows together, from the pow of 2 all the way up to the pow of 5300. You now have a Very Large Number.

**QUESTION:**
What is the largest whole number q such that 2010 to the qth power divides that Very Large Number (with no remainder)?

#4 - THREE CUBES

Quick review: To cube a number, you take the number times itself, then times itself again. So, for example, two cubed is two times two times two, which is eight.

I’m thinking of three whole numbers. If I cube each of them, and then add the results together, I get three. Of course, I’m thinking of the numbers one, one, and one.

**QUESTION:**
What is a different set of three numbers that also does this? (In other words, you’re looking for a set of three whole numbers whose cubes add up to three, other than the set (1,1,1).)
#5 - PENTAPRIMES

Let’s make fractions out of prime numbers by taking their reciprocals. In other words, divide the number one by various primes, like this: \( \frac{1}{3}, \frac{1}{7}, \frac{1}{11} \).

If you write them in decimal form, you often get repeating decimals:

- \( \frac{1}{3} \) is .3 repeating (in other words .33333…);
- \( \frac{1}{7} \) is .142857 repeating (in other words, .142857142857…);
- \( \frac{1}{11} \) is .09 repeating (.09090909…).

Note that the number of digits that repeats is different in each of these three cases: 1/3 has one repeating digit, 1/11 has two, and 1/7 has six.

Note also that each is purely periodic, which means that everything that comes after the decimal point is part of the repeating pattern.

**QUESTION:**

Find each of the two prime numbers whose reciprocal is purely periodic with exactly five repeating digits.

(In other words, you are looking for the two different primes \( p \) for which the decimal representation of \( 1/p \) repeats immediately after the decimal point, and with exactly five repeating digits, over and over.)

#6 - ...AND THAT’S ANOTHER HUNDRED

Take a sequence of three positive whole numbers that add up to 100. For example, you might take 40, then 30, then 30. (Or, you might take 30, then 30, then 40, which is, of course, a different sequence of numbers, since for sequences, the order matters.)

**QUESTION:**

How many such sequences of numbers exist?

(Remember, we’re looking for sequences of positive, whole numbers, which means that zero doesn’t count.)

#7 - O.D.D.

Take a number, and count up its one-digit divisors. So, for example, take 100. It’s divisible by 1, and by 2, and by 4, and by 5. Those are its one-digit divisors, and there are four of them. Let’s call this a D-number, so we’ll say that 100’s D-number is 4.

Similarly, 4’s D-number is 3, and 1’s D-number is 1, and so on.

**QUESTION:**

All the whole numbers from 1 to 100 have D-numbers, of course. What do you get when you add them all up?